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# Low Energy Segmentation (a first look)

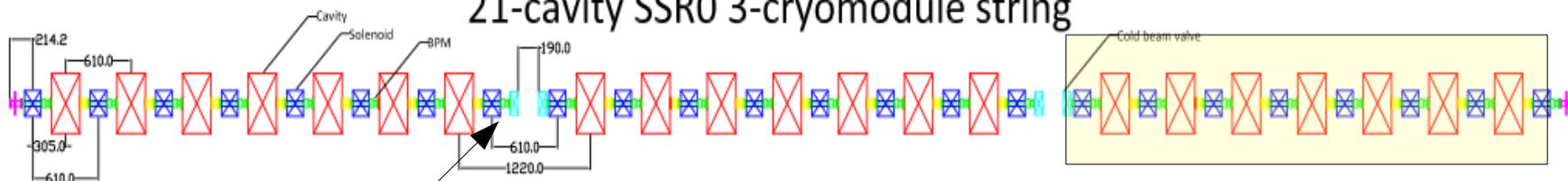
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APC



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- **Warm space is needed for collimation and possibly also for instrumentation.**
  - Taking advantage of the high gradient in SC cavities at low energy necessarily leads to a compact arrangement of cavities and focusing elements with no longitudinal space.
  - The “baseline” design for the SSR0 section assumed 18 cells in a single cryostat.
  - Opening longitudinal space is especially problematic at low energy because of the short period and space charge defocusing.
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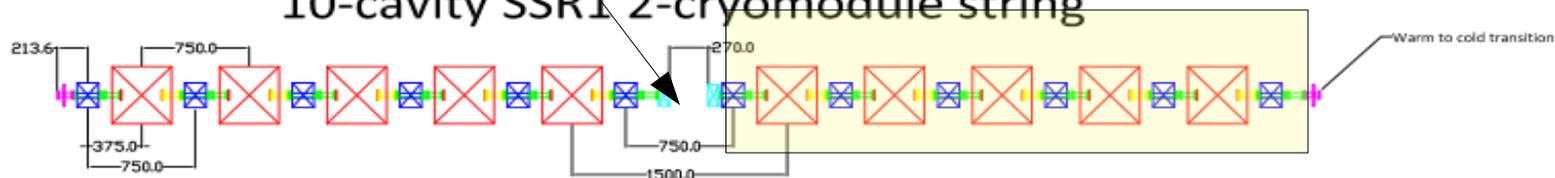


21-cavity SSR0 3-cryomodule string



Warm insertion ~ Periods with missing cavity.

10-cavity SSR1 2-cryomodule string





We assume  $\varepsilon$  remains constant, so beam size is proportional to  $\sqrt{\beta}$ .  
Neglecting space charge, the (longitudinal)  $\beta$  function in a drift obeys the relation:

$$\beta(z) = \beta^* + \frac{(z - z^*)^2}{\beta^*} \quad z^* = \text{Waist position}$$

Let  $z^* = 0.0$ , and  $-L_1, L_2$  the upstream and downstream end of the drift space. For a fixed value of  $L_1$  the minimum value of the  $\beta$  function at the upstream extremity occurs when

$$\frac{d\beta}{d\beta^*} = 1 - \frac{L_1^2}{\beta^{*2}} = 0 \quad \beta^* = L_1 \quad \beta = 2L_1$$

And similarly, at the downstream extremity. Clearly, a minimal extreme  $\beta$  for the drift region is obtained when  $L_1 = L_2 = L$ . In that case,

$$\frac{\beta(\pm L)}{\beta^*} = \frac{2L}{L} = 2 \quad \text{and thus} \quad \frac{\sigma_z(\pm L)}{\sigma_z^*} = \sqrt{2}$$



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Within a “period” with no longitudinal focusing (i.e. missing cavity), the minimum achievable  $\beta$ -function at the extremities is  $2L$ , where  $L$  is the distance to the waist. Since the emittance is fixed, this sets the transverse beam size.

Note that **only** when the optimal  $\beta$  at the extremities is reached, does one have  $\sigma/\sigma^* = \sqrt{2}$ .

**This provides a prescription for longitudinal matching:**

- (1) drop the field in the next to last cavity
- (2) Iterate until the beam size in the last cavity is near the theoretical optimum
- (3) adjust the field in the last cavity to get a waist in the center

# What About Space Charge ?



In the presence of space charge, the rms envelope equation in a drift takes on the form

$$R'' - \frac{K}{R} - \frac{\epsilon}{R^3} = 0 \quad \text{With} \quad R = \sqrt{\beta\epsilon}$$

where  $K > 0$  depends on the charge density. Rearranging a bit

$$R'' - \frac{KR^2 + \epsilon}{R^3} = R'' - \frac{\hat{\epsilon}}{R^3} = 0 \quad \hat{\epsilon} > \epsilon$$

In the first approximation, the effect of space charge is more or less like that of an increase in emittance. It does not change the optimal ratio  $\sigma/\sigma^* = \sqrt{2}$ , but it increases the maximum beam size to  $\sqrt{\beta\hat{\epsilon}}$



- $|\varphi_s|/\sigma_s$  should remain more or less the same before and after introduction of the drift space, so as to preserve the longitudinal acceptance ( “bucket” width  $\sim 3 \times |\varphi_s|$  ).
- This implies reducing the field in the cavities on both sides of the insertion to reduce rf focusing, resulting in a loss of acceleration. With 3 cryostats x 7 periods, we gained 3 cavities w/r to baseline however, 6 cavities have to operate at reduced voltage.
- It is difficult to regain the loss in acceleration by raising the cavity field in the last SSR0 periods. Even if the max allowed peak surface magnetic field in the cavity (60-75 mT) is not reached, another limit is set by the max allowable solenoid field (  $\sim 6-6.5$  T).
- Changing the energy at the input of the next section (SSR1) is a significant perturbation on its optics. This is certainly possible; it is also a time-consuming exercise and may affect the optimization of the transitions between sections i.e. cavity families.
- Transverse phase advance between the 2 insertions should be  $\sim \pm 90$  deg



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- In the limited time we had, we looked primarily at SSR0, as this is the most problematic area.
  - Matching is done using TraceWin. The code is geared toward matching between long regular sections with many periods.
  - In this case, manual intervention is required. An important issue is the fact that we need to fix the synchronous phase profile. The code currently cannot hold the synchronous phases constant during matching iterations.
  - The matching procedure involves changing the field in the cavities in succession, and resetting the input phases to keep the synchronous phases fixed after each change.
  - Longitudinal matching is performed first. Once an acceptable solution has been reached longitudinally, the solenoids can be adjusted to get a regular transverse envelope.



$$\epsilon_n = 0.275 \quad \pi \text{ mm-mrad (normalized longitudinal emittance)}$$

$$\epsilon = \epsilon_n / \beta\gamma^3 \quad \pi \text{ mm-mrad (unnormalized longitudinal emittance)}$$

At the first insertion  $\beta\gamma^3 = 0.0963(1.004672)^3 = 0.0976$

Distance between the exit of upstream cavity and entrance of downstream cavity ~ 520 mm

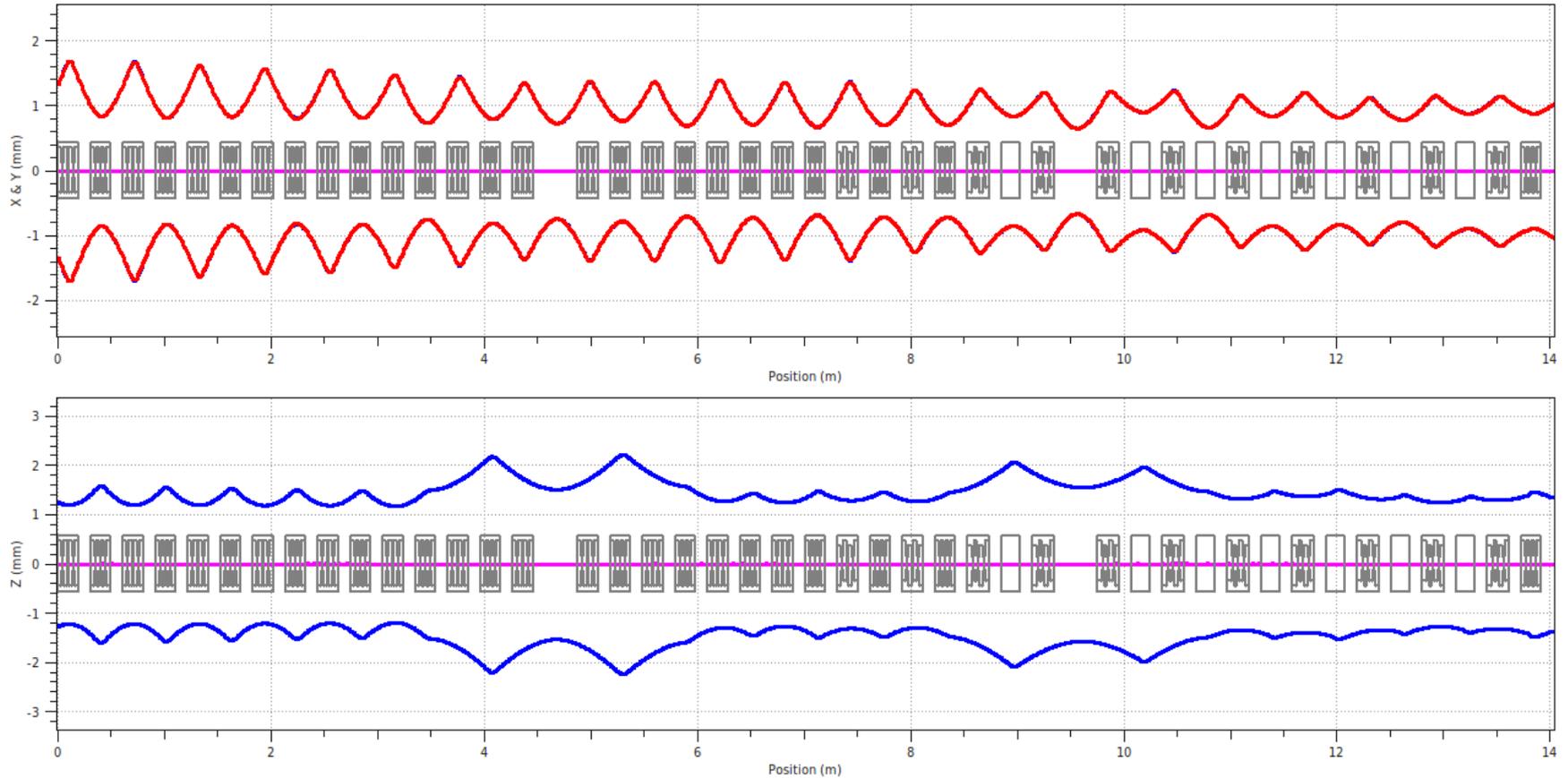
$$\sigma^* = \sqrt{\beta^* \epsilon} \simeq \sqrt{520 \times 2.818 \times 10^{-3}} = 1.2105 \quad \text{mm}$$

$$\beta(\pm L) = \sqrt{2} \times 1.21 = 1.69 \quad \text{mm}$$

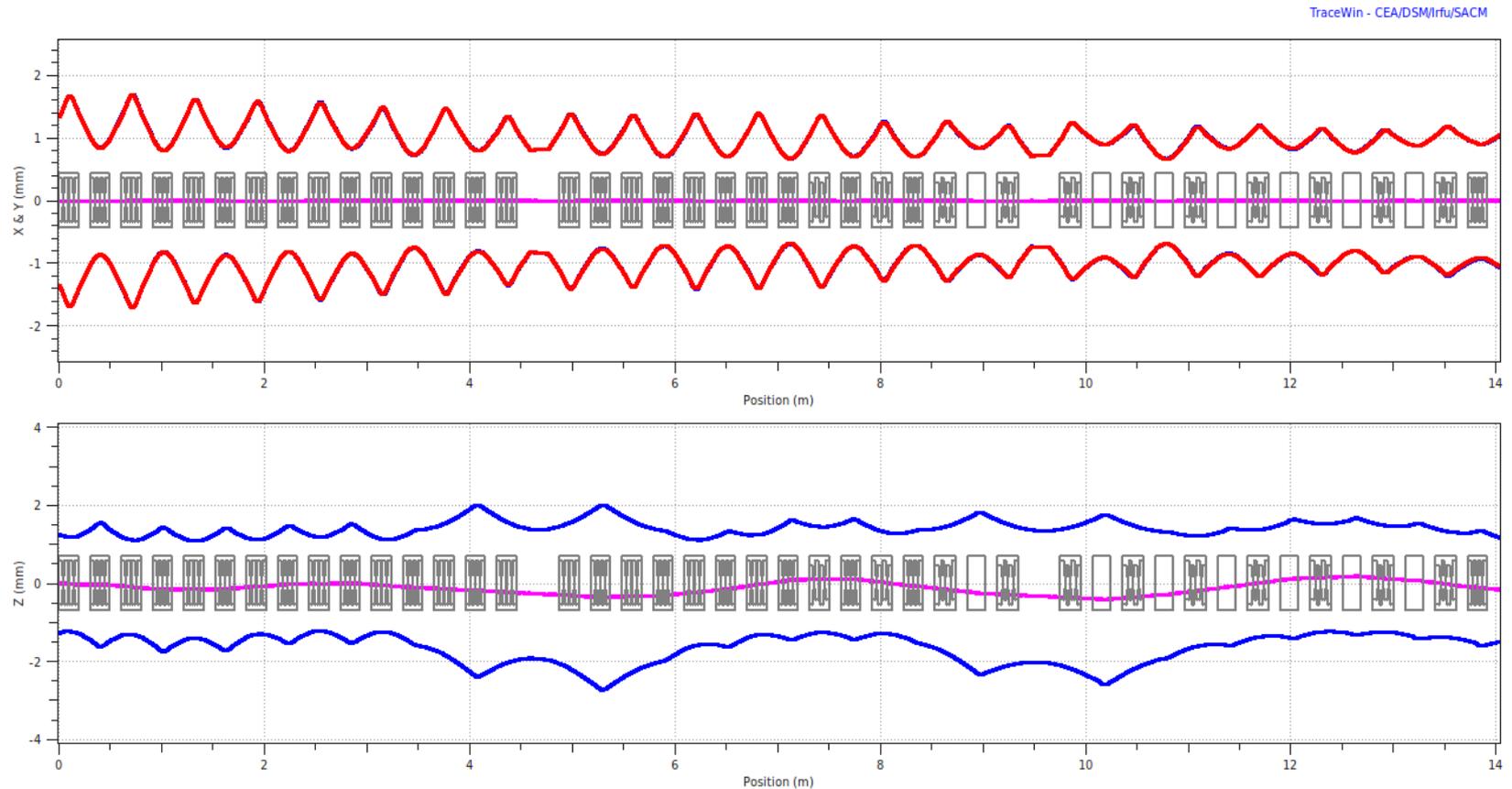
# Matched Solution (envelope calculation)

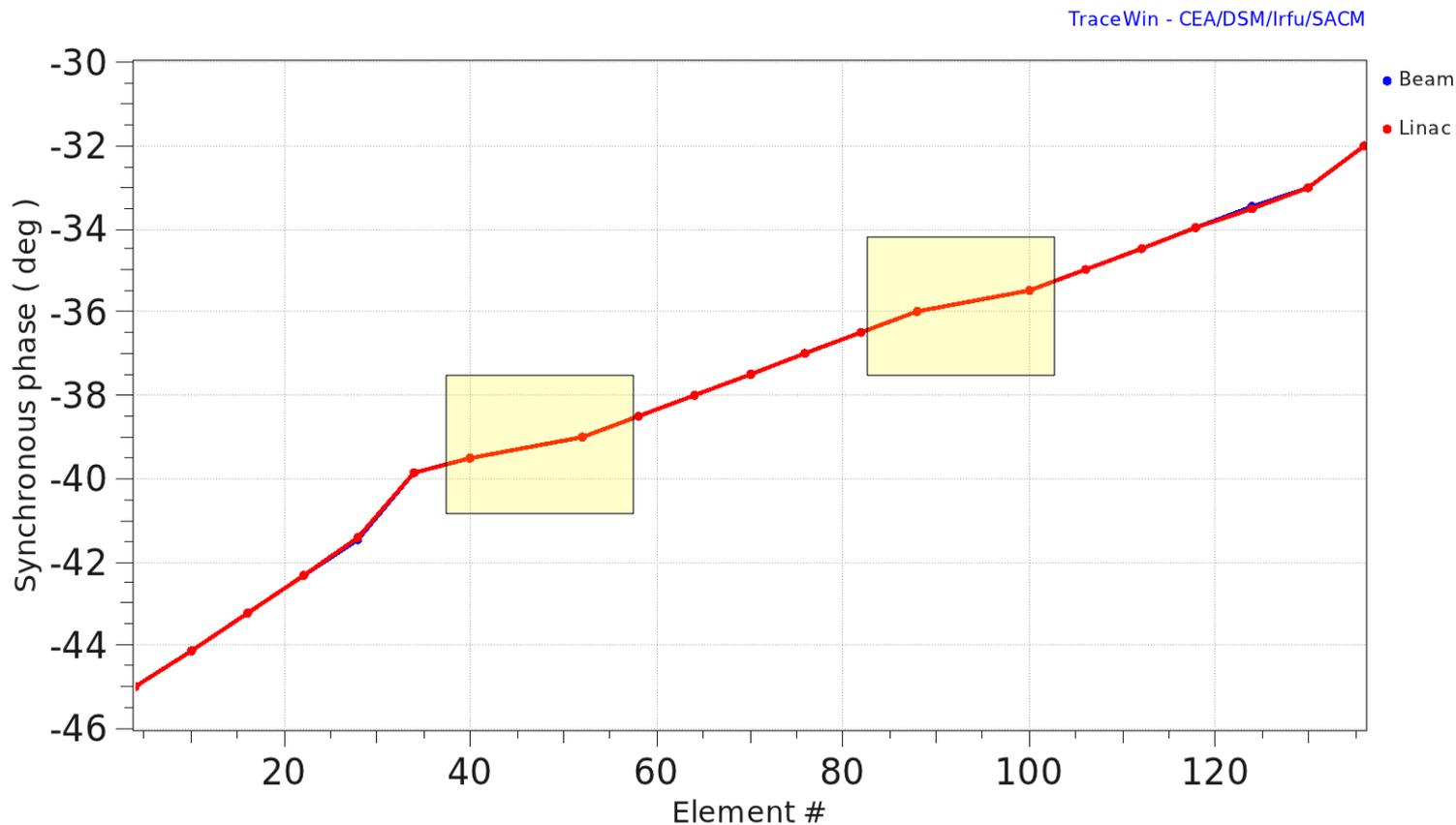


TraceWin - CEA/DSM/Irfu/SACM



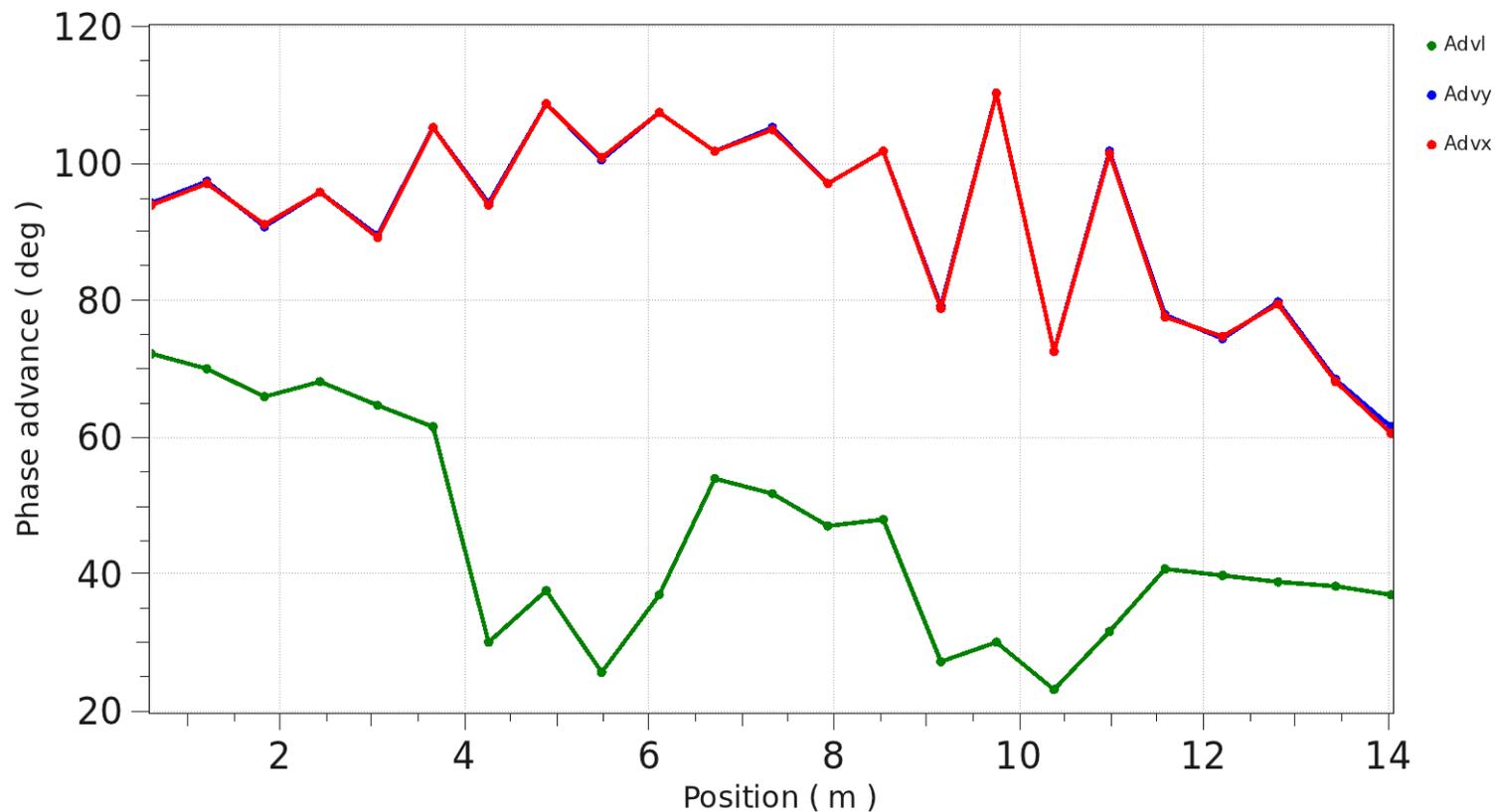
# Matched Solution (Tracking)





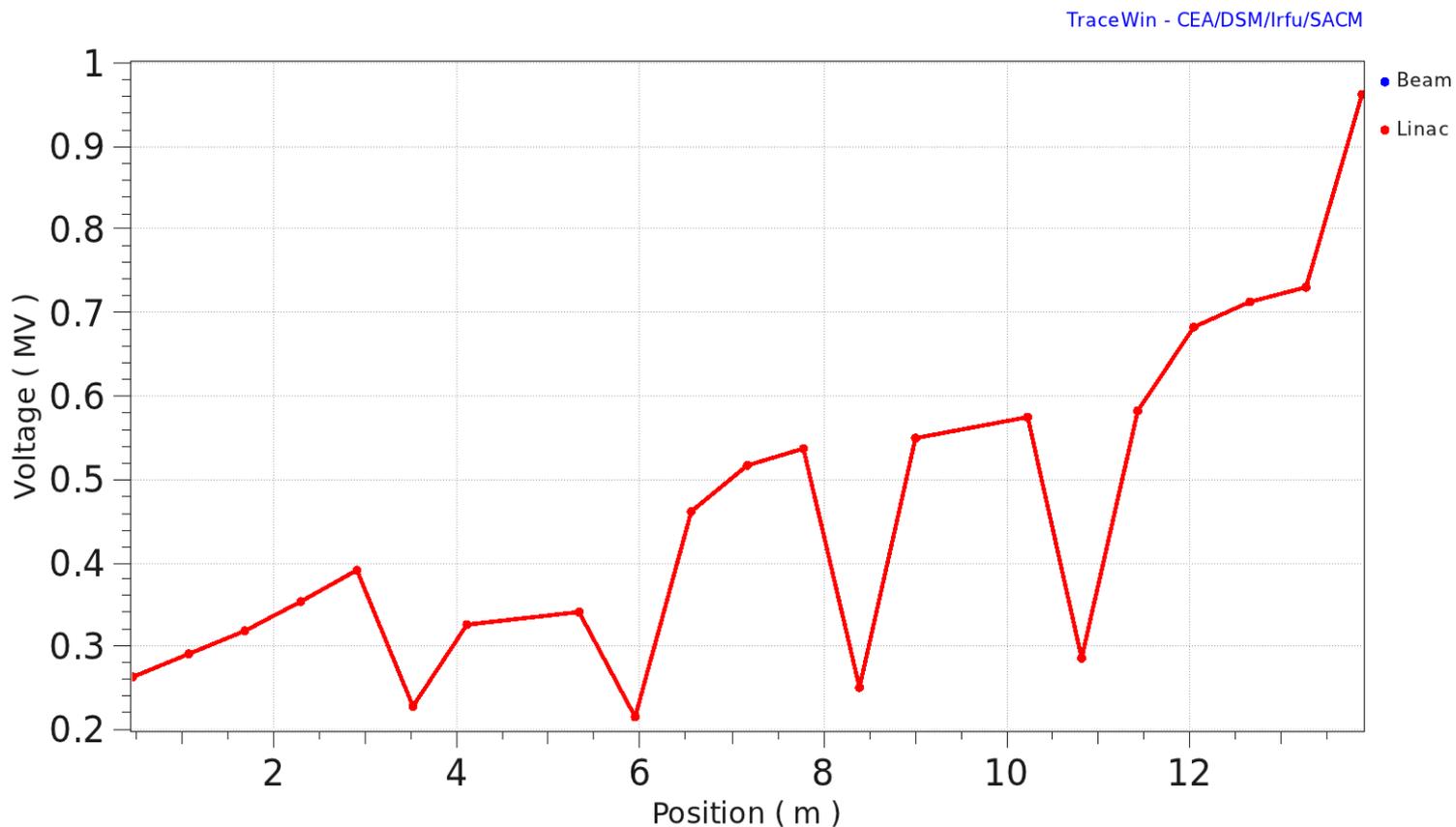


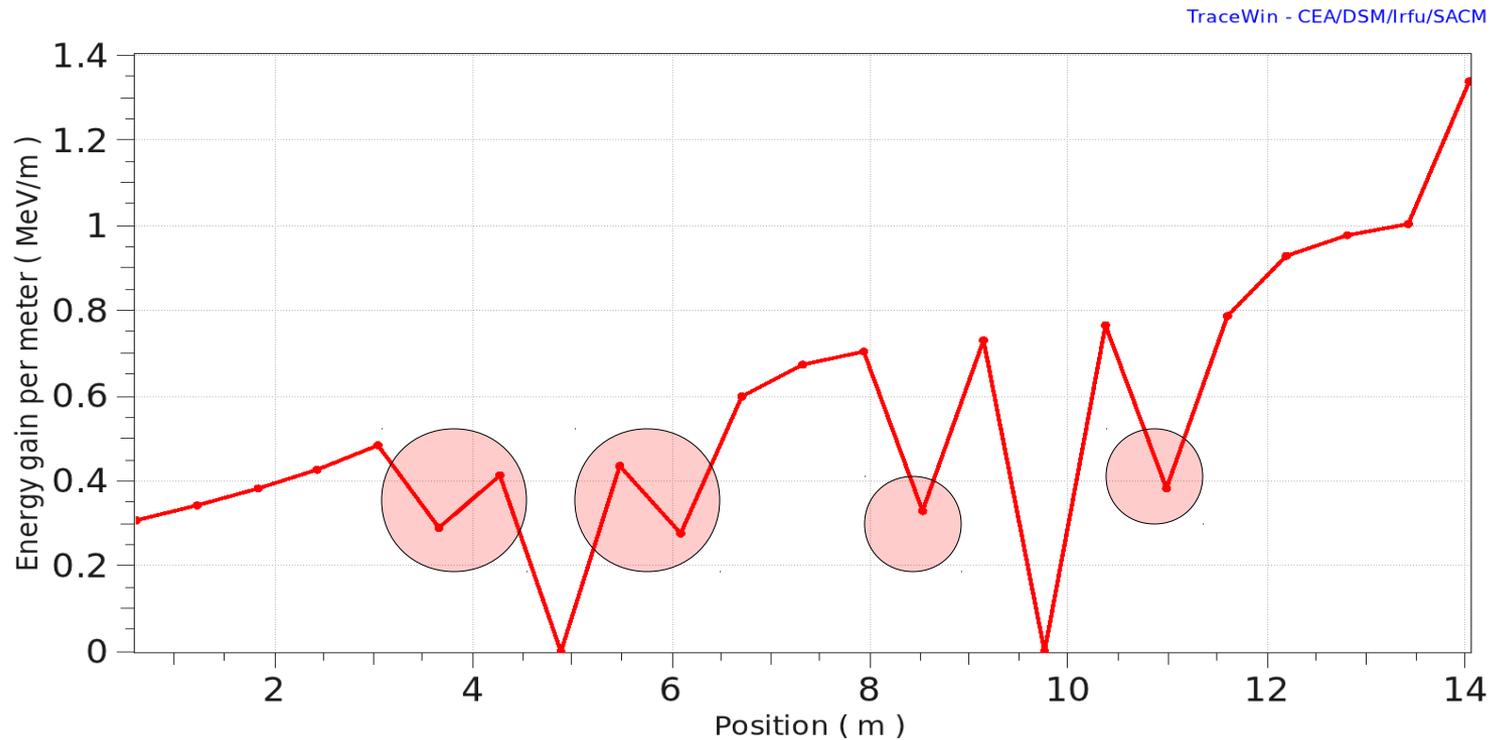
TraceWin - CEA/DSM/Irfu/SACM





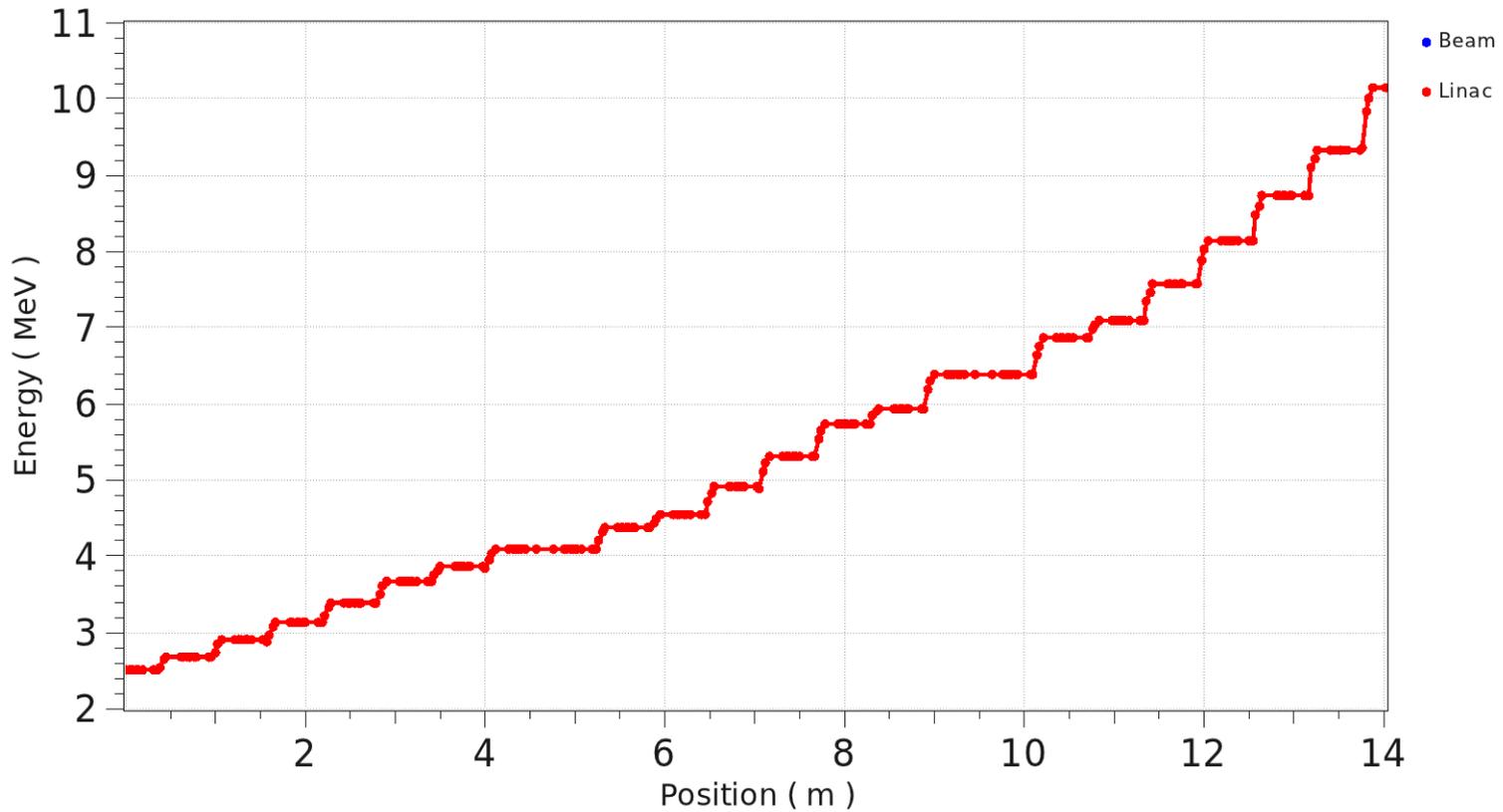
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- The transverse phase advance needs to be further optimized.
  - It should be possible to make it vary monotonically from  $\sim 90$  to  $\sim 60$  deg.
  - It should also be possible to ensure a  $\sim 90$  deg phase advance between the 2 insertions (beam phase advance with sc).
  - This has not been done yet.







TraceWin - CEA/DSM/Irfu/SACM





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- “Long” open insertions are difficult at low energy because of the inherently short lattice period.
  - The “design” presented here is a first iteration and is certainly not the last word. The transverse phase advance needs to be reduced and smoothed. In the last periods, the cavity and solenoid fields are a bit over the limit.
  - Nevertheless, the matching procedure has been established and we understand where the difficulties lie.
  - Any issue that might result in lattice modifications at low energy have a potentially significant impact on the entire optical design.
  - It is important that we understand exactly how much longitudinal space will be needed to accommodate collimation and/or instrumentation (e.g. profile monitors).