MI departmental note
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A parallel-plate capacitor as a model of a kicker

A.Shemyakin, L. Prost

Electrostatic kicker and unit length capacitance

The simplest model of an electrostatic kicker is two parallel plates infinite in one direction (chosen as Y in Fig.1).

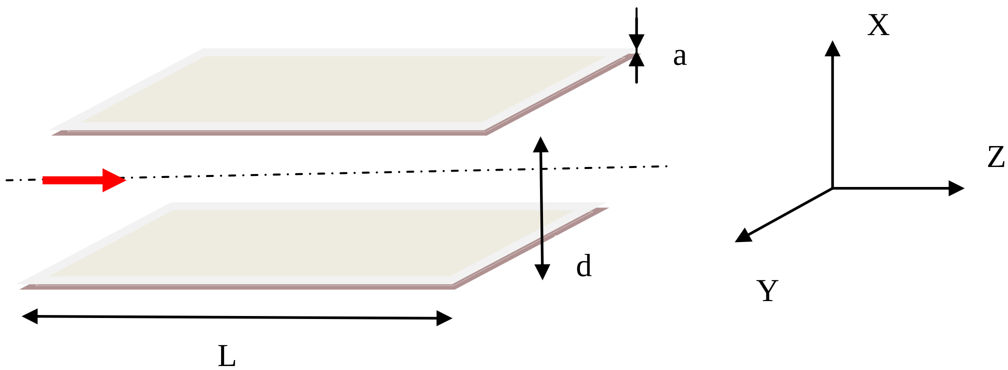

Figure 1. Plate geometry and dimensions.

A particle with a charge e traveling between two plates in Z direction acquires a momentum in X direction equal to

$$p_x = \int_{-\infty}^{\infty} eE_x(z)\frac{dz}{v_z}, \tag{1}$$

where E_z is plates' electric field and v_z is the particle velocity. In the approximation that the particle velocity change and its trajectory deviation from a straight line are negligible, the integral is proportional to the flux through a plane between plates and, through the Gauss theorem, to the line charge ρ of the plate:

$$p_x = \frac{e}{v_z}\int_{-\infty}^{\infty} E_x(z)dz = \frac{e}{v_z}\frac{\rho}{\varepsilon_0} \tag{2}$$

The line charge can be expressed through the capacitance C of the unit length (in Z direction) and the voltage between plates U

$$p_x = \frac{e}{v_z}\frac{C}{\varepsilon_0}\cdot U \tag{3}$$

Capacitance of two parallel plates

Electric field between tow plates was simulated with SAM code [1] for $U = 20$ kV, $a = 1$ mm, $d = 20$ mm, and $L = 1 \div 100$ mm. The distributions of the electric field E_x along the central plane are presented in Fig.2.

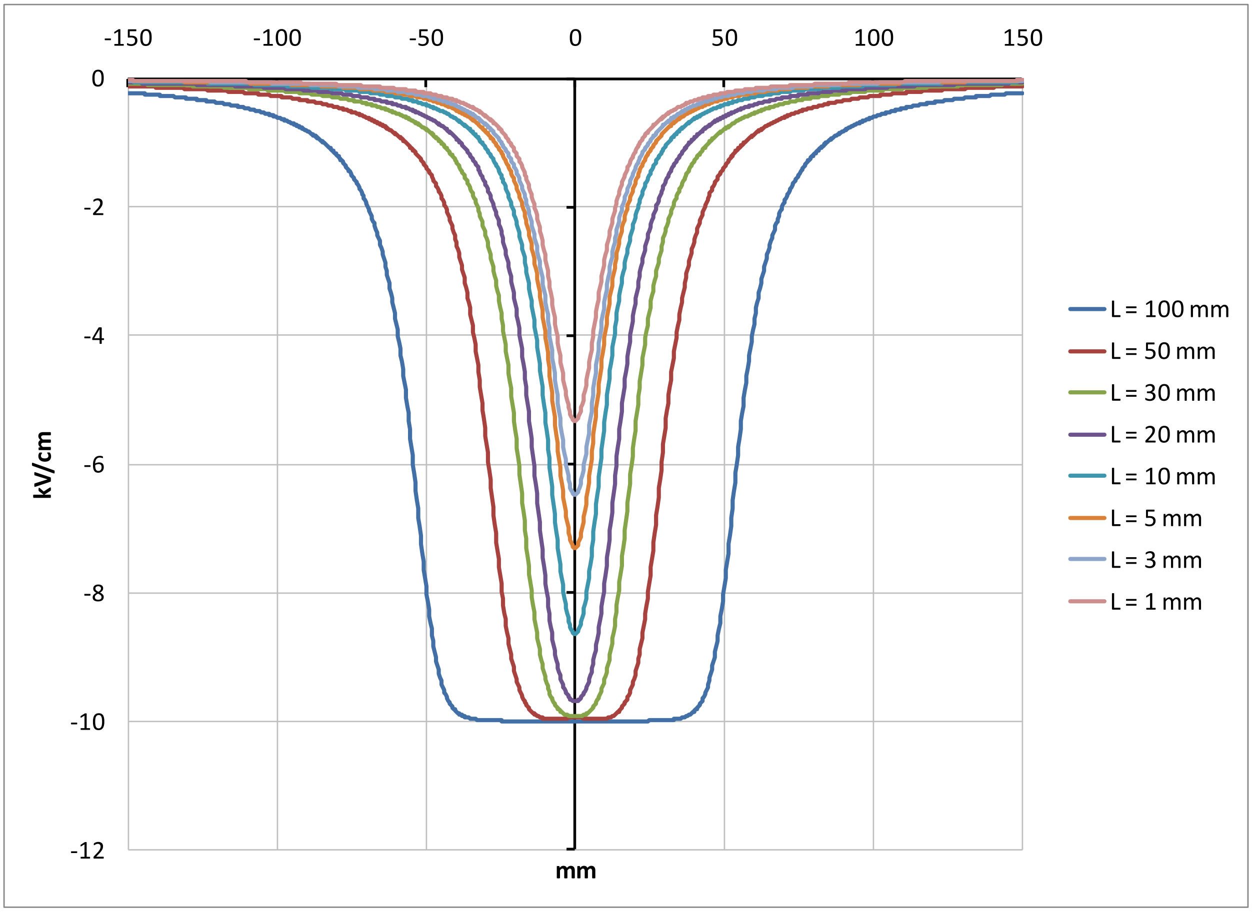

Figure 2. Electric field distribution along the central plane ($X=0$).

The capacitance calculated from the integrals over 300mm and capacitance directly calculated by SAM itself are shown in Fig.3. Capacitance calculated from the integrals and normalized by ε_0 is presented in Fig.4. The solid line is a simple approximation adjusted to fit points with large L/d

$$C=\varepsilon_0\left(\frac{L}{d}+k\right) \tag{4}$$

at $k=1.2$. Simulations in Ref. [2] are approximated by Eq.(4) with $k=1.01$ without indication of the plate thickness.

Note that the point with the smallest value of L, 1mm, corresponds to two square wires. One can approximate this case by two round wires of radius $a=0.5$mm with $d=20$ mm between centers. Unit length capacitance of two round wires can be calculated analytically [3]

$$C_w=\frac{\pi\varepsilon_0}{\ln\left(\frac{d}{2a}+\sqrt{\left(\frac{d}{2a}\right)^2-1}\right)} \tag{5}$$

For $2a/d=0.05$, Eq. (5) gives $C_w/\varepsilon_0=0.85$, the value close to the one found in the simulations $C(L=1mm)/\varepsilon_0=0.84$. To some extent, one can consider the capacitance of the parallel plate capacitor as a sum of the capacitance of the flat portion

$$C_f = \varepsilon_0 \frac{L}{d} \tag{6}$$

and a contribution of the edge effects, which can be approximated by a couple of wires:

$$C \approx C_f\left(\frac{L}{d}\right) + C_w\left(\frac{d}{a}\right) \tag{7}$$

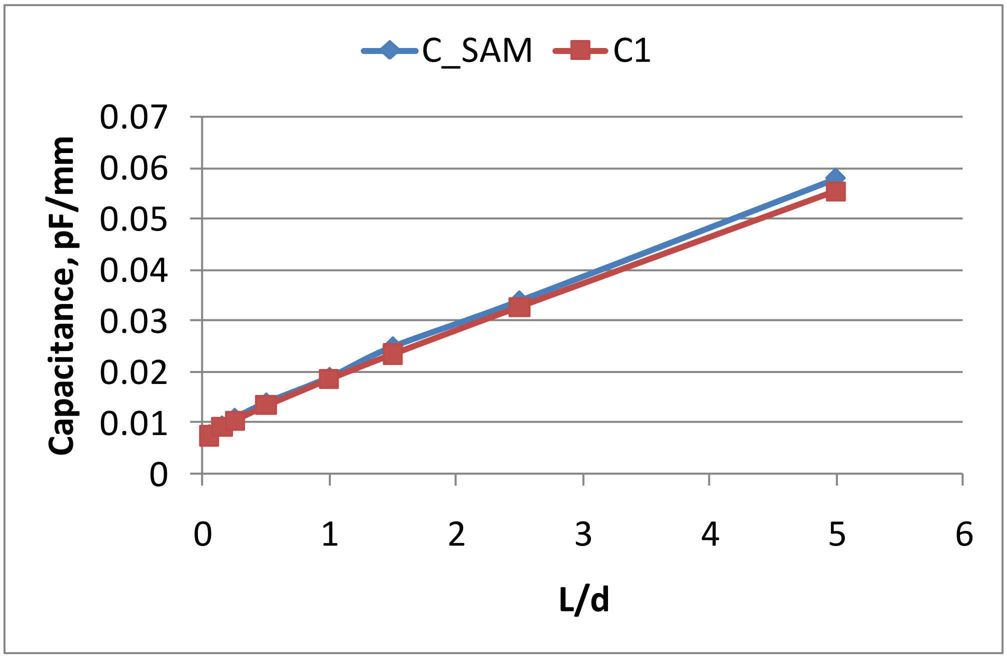

Figure 3. Capacitance calculated from the data of Fig.2 (labeled C1) and directly by SAM.

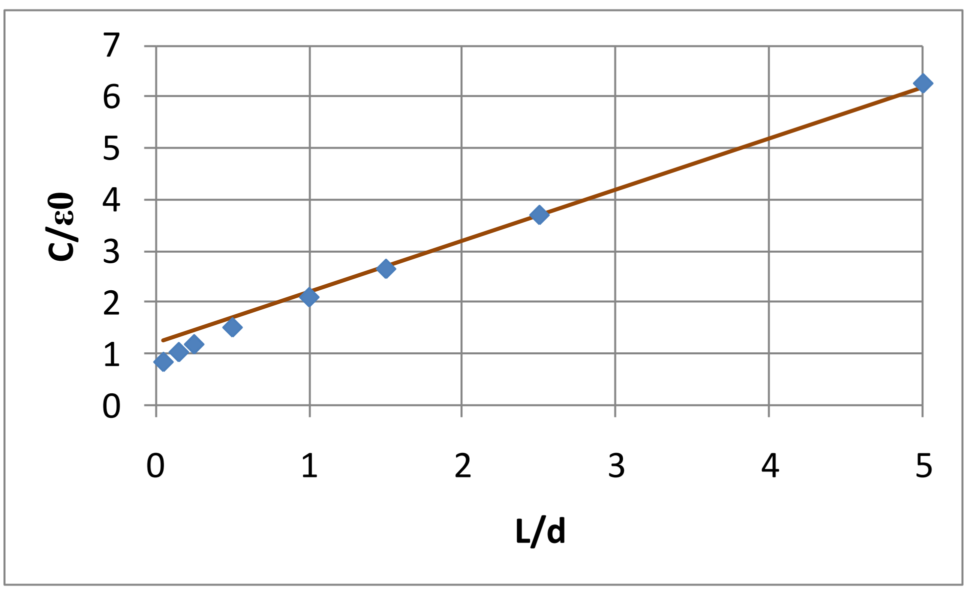

Figure 4. Capacitance C1 from Fig.3 normalized by ε_0 (data points) and the approximation by Eq.(4) (solid line).

Figure 5 presents two more parameters again as functions of the L/d ratio. The first is the value of the maximum electric field normalized by U/d. The second parameter characterizes the width of the electric field distribution along the central plane and is equal to the total length where the field amplitude is above 10% of its maximum for the given L/d.

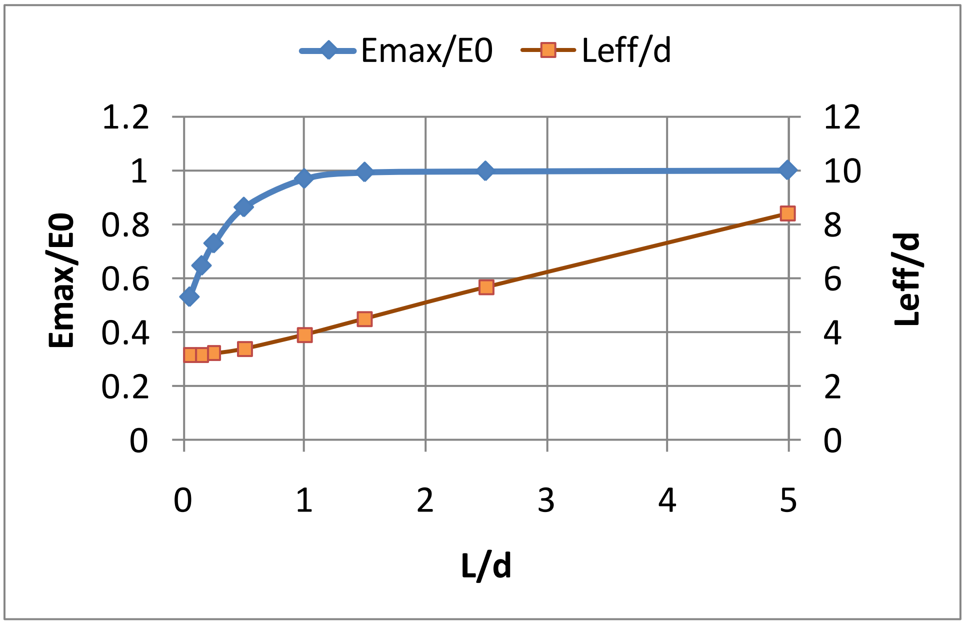

Figure 5. Dependence of the maximum electric field and the length of area with the electric field above 10% of its amplitude on the L/d ratio.

Discussion

1. If one would like to minimize the effective length of an electrostatic kicker at a given gap while preserving as much as possible the kick efficiency, the optimum seems to be at the length of the plates close to the gap size.
2. In the used approximation of small angles and constant longitudinal velocity, the total kick doesn't depend on the (parallel) offset of the trajectory by the nature of derivation of Eq.(3). It means that for small angles there are no aberrations at any shape of the electrodes, and optimization of a travelling wave kicker can be done primarily from the point of view of optimizing an impedance, electrode coupling etc..

1. B. Fomel et al., Preprint Budker INP 96-11, 1996
2. S. Catalan-Izquierdo et al., ICREPQ'09, http://www.icrepq.com/ICREPQ'09/451-izquierdo.pdf
3. http://en.wikipedia.org/wiki/Capacitance