Intrabeam Stripping Loss Estimates For Project-X

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Acknowledgments

SNS for generously providing access to their experimental results

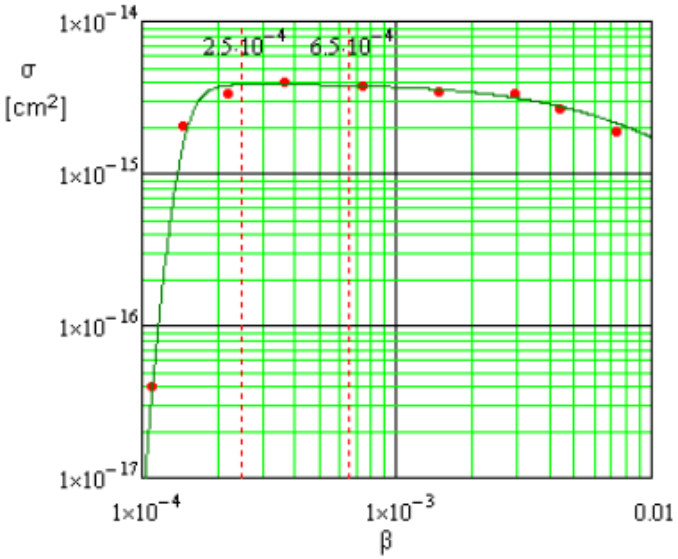
Valeri Lebedev for proposing that we investigate intrabeam stripping as a possible loss mechanism and for introducing the basic model.

Many others, (Solyak, Yakovlev, Nagaitsev ...) for going through the derivations.

Preliminaries

- **V. Lebedev** proposed intra-beam stripping as a mechanism to account for unexplained losses at SNS.
- He performed estimates for both SNS and PrX
- His initial results suggested that we need to pay attention.
- Valeri used a simplified MATCAD model to produce his estimates.
- I wrote a program to produce an estimate for Project-X using as input, the data generated by TraceWin/PARTRAN (CEA/Saclay tracking code)
- The IBS formula has been cross-checked by a number of people; after a few iterations, every one agrees.

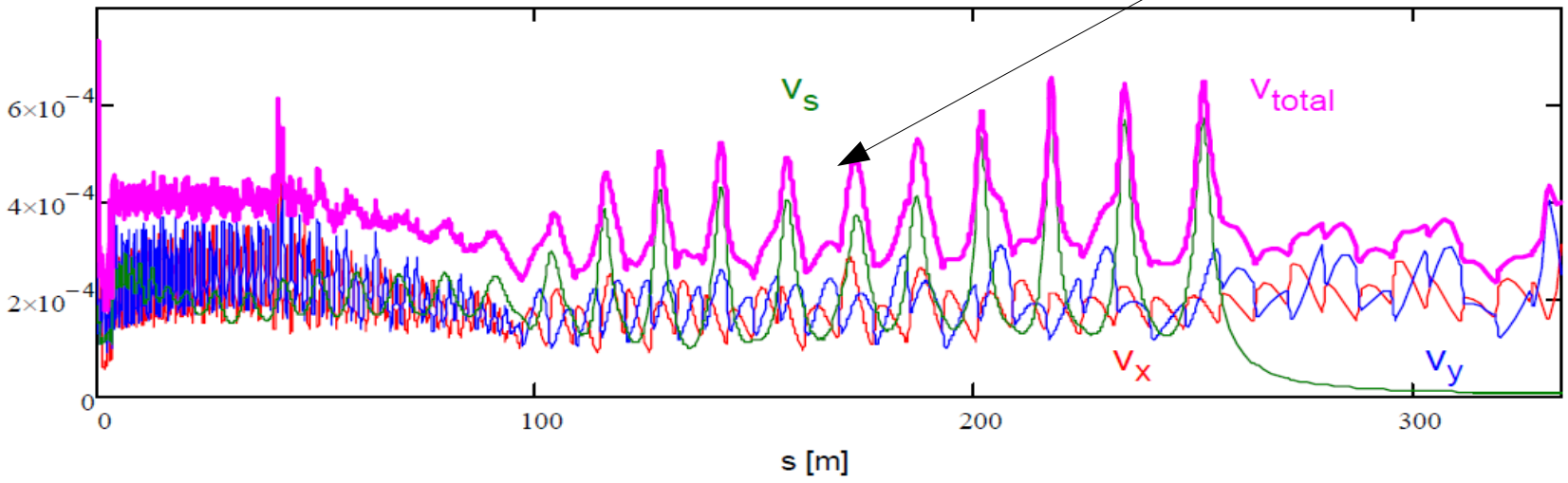
H- Stripping Cross -Section



Stripping cross-section is essentially constant for relative velocities in the range

$$2.0\times10^{-4} < \beta < 3 \times 10^{-3}$$

Velocities spreads are expected to be within that range. For **SNS, Valeri's early estimate is shown below.**



Stripping Rate

Fractional particle losses per unit of distance:

Beam sizes and velocity spreads are measured in the lab frame

$$\frac{1}{N}\frac{dN}{ds} \simeq \frac{N\sigma_{\text{stripping}}}{8\pi^2\sigma_x\sigma_y\sigma_s\gamma^2}\sqrt{\gamma^2\bar{\theta}_x^2+\gamma^2\bar{\theta}_y^2+\bar{\theta}_{\|}^2}$$

Assumes: Gaussian distribution and $\sigma_{\text{stripping}}$ = constant

$\sigma_x, \sigma_y, \sigma_s$ Transverse and long rms beam sizes in the lab frame

$\bar{\theta}_x^2, \bar{\theta}_y^2, \bar{\theta}_{\|}^2$ Transverse and long mean squared relative velocity spreads, in the lab frame.

$\sigma_{\text{stripping}}$ Total Intrabeam stripping cross-section

Derivation

In the bunch reference frame, the **scattering** probability for per unit time for an ion is:

(Assuming velocities are not relativistic in BF !)

$$-\left.\frac{1}{N}\frac{dN}{dt}\right|_{\mathrm{BF}} = \frac{1}{N}\int d\mathbf{r}\int d\mathbf{v}_1 N f(\mathbf{r},\mathbf{v}_1)\int d\mathbf{v}_2 N f(\mathbf{r},\mathbf{v}_2)|\mathbf{v}_1-\mathbf{v}_2|\frac{d\sigma}{d\Omega}d\Omega$$

$$-\left.\frac{1}{N}\frac{dN}{dt}\right|_{\mathrm{BF}} = \sigma N\int d\mathbf{r}\int d\mathbf{v}_1 f(\mathbf{r},\mathbf{v}_1)\int d\mathbf{v}_2 f(\mathbf{r},\mathbf{v}_2)|\mathbf{v}_1-\mathbf{v}_2|$$

The stripping rate is ½ of the scattering rate, since for each scattering event experienced by a given ion, the probability that it looses an electron is ½.

Gaussian Distributions

- We assume stationary distributions in all 3 phase space plane
- We assume equilibrium MB-distribution with independent "temperatures" in each plane (the invariant emittances play the role of the energy).
- For e- or e+ rings: random photon emission + central limit theorem imply equilibrium is quickly reached and distributions are truly Gaussian.
- For a proton linac, equilibrium may not be reached … so the Gaussian distribution assumption involves some hand waving

For x-x' (and similarly for the other planes):

$$f_x(\epsilon_x) = f_x(\epsilon_x(x,x')) = f_x(x^2 + (-\alpha x + \beta x'^2))$$

$$f_x(x,x') = \frac{1}{2\pi\epsilon_x}\exp(-\frac{1}{2\beta\epsilon_x}(x^2 + (-\alpha x + \beta x')^2))$$

$$= \frac{\beta}{2\pi\sigma_x^2}\exp(-\frac{1}{2\sigma_x^2}(x^2 + (-\alpha x + \beta x')^2))$$

Local Velocity Distribution

$$f(x,x') = \frac{1}{2\pi\epsilon_x}\exp\left\{-\frac{\beta^{-1}}{2\epsilon_x}(x^2+(-\alpha x+\beta x')^2)\right\}$$

Remarkably, for a "Gaussian" distribution in phase space, the velocity distribution **at a given x** has the **same spread at any x.**
This can be seen by rearranging the above as follows:

$$f(x,x') = \frac{1}{2\pi\epsilon_x}\exp\frac{-x^2}{2\epsilon_x\beta}\exp\frac{-(x'-\frac{\alpha}{\beta}x)^2}{2\epsilon_x/\beta}$$

The velocity distribution at a location x has an average: $\bar{x'} = -\frac{\alpha}{\beta}x$

And a standard deviation: $\sqrt{\epsilon_x/\beta}$

Local vs Total vs Velocity Spread

It is worth reiterating that what matters for IBS is the local **velocity spread.** This is **not the same** as the total velocity spread i.e. the spread computed over the entire extent of the beam.

Local velocity (angular) spread: $\sqrt{\epsilon_x/\beta}$ ← **This is what matters**

Total velocity (angular) spread:

$$< x'x' >= \frac{\epsilon_x^2 + < xx' >^2}{< xx >} = \frac{\epsilon_x^2(1+\alpha^2)}{\beta\epsilon_x}$$

$$\sigma_{x'} = \sqrt{\gamma\epsilon_x}$$ **...not this**

Stripping Probability in the Lab Frame

$dt_L = \gamma dt_B$ Relativistic transformation of time between frames

$$-\frac{1}{N}\frac{dN}{\beta c dt_L} = \frac{1}{N}\frac{dN}{ds}\bigg|_{\mathrm{LF}} = \frac{\sigma N}{2\beta c\gamma}\int d\mathbf{r}\int d\mathbf{v}_1 f(\mathbf{r},\mathbf{v}_1)\int d\mathbf{v}_2 f(\mathbf{r},\mathbf{v}_2)|\mathbf{v}_1 - \mathbf{v}$$

The integral is in the beam frame

Assuming a separable Gaussian spatial distribution, we can integrate over dr

$$-\frac{1}{N}\frac{dN}{ds}\bigg|_{\mathrm{LF}} = \frac{\sigma N}{2(4\pi)^{3/2}\gamma^2\beta c\sigma_x\sigma_y\sigma_z}\int d\mathbf{v}_1 f(\mathbf{v}_1)\int d\mathbf{v}_2 f(\mathbf{v}_2)|\mathbf{v}_1 - \mathbf{v}_2|$$

The integral is in the beam frame

In the LAB frame

An extra factor γ has been introduced due to the relativistic transformation from z_B to z_L

After Velocity Integration

with $u = |\mathbf{v_1} - \mathbf{v_2}|$
$v = |\mathbf{v_1} + \mathbf{v_2}|$

Integration is straightforward in the "isotropic" case i.e. when the T in each plane are equal

$$-\frac{1}{N}\frac{dN}{ds}\bigg|_{\mathrm{LF}} = \frac{\sigma N}{2(4\pi)^{3/2}\gamma^2\beta c\sigma_x\sigma_y\sigma_z} < v_{\mathrm{BF}} >$$

velocity spread in beam frame

$$-\frac{1}{N}\frac{dN}{ds} \simeq \frac{N\sigma_{\mathrm{stripping}}}{8\pi^2\sigma_x\sigma_y\sigma_s\gamma^2\beta c}\sqrt{\sigma_{v_x}^2 + \sigma_{v_y}^2 + \sigma_{v_z}^2}\cdot F(\theta_x, \theta_y, \theta_z)$$

F is a form factor which is =2/√3 (max) when all 3 velocity spreads are equal.
F varies by less than 20% and depends only on the ratios of its arguments.

Expressed in terms of LAB frame angles:

$$\sigma_v|_{\mathrm{BF}} = \beta c\sqrt{\gamma^2\theta_x + \gamma^2\theta_y^2 + \theta_z^2}$$

Velocity Addition Refresher

General

$$\vec{\beta}_1 \oplus \vec{\beta}_2 = \frac{\vec{\beta}_1 + \vec{\beta}_{2\parallel} + \frac{1}{\gamma}\vec{\beta}_{1\perp}}{1 - \beta_1\beta_2}$$

With $\beta_1 \simeq \beta_2 = \beta$

$$(\vec{\beta}_1 \oplus \vec{\beta}_2)_\perp \simeq \frac{\frac{1}{\gamma}\vec{\beta}_\perp}{1-\beta^2} = \gamma\vec{\beta}_\perp$$

Valid for $\frac{\Delta\gamma}{\gamma} << 1$

$$(\vec{\beta}_1 \oplus \vec{\beta}_2)_\parallel = \frac{\beta_2 - \beta_1}{1 - \beta_1\beta_2} \simeq \beta_1\gamma^2 \left\{ \left(1 + \frac{2\Delta\gamma}{\beta_1\gamma^3}\right)^{1/2} - 1 \right\}$$

$$\simeq \frac{\Delta\gamma}{\beta\gamma} \simeq \beta\frac{\Delta p}{p}$$

Angle Variables

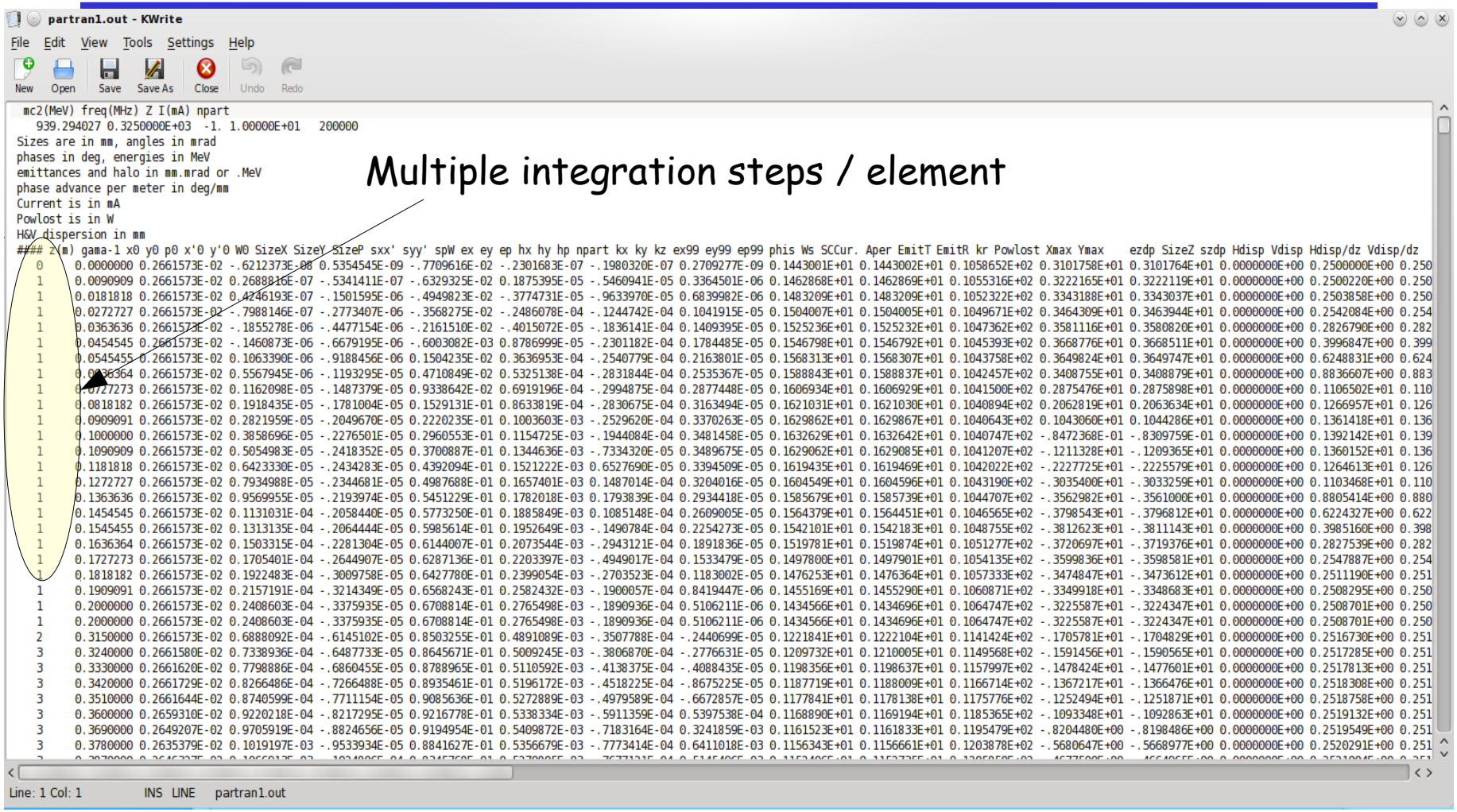
In the lab frame:	In the beam frame
$\theta_x = \frac{v_x}{\beta c} = \frac{\beta_x c}{\beta c} \simeq x'$	$\gamma\theta_x$
$\theta_y = \frac{v_y}{\beta c} = \frac{\beta_y c}{\beta c} \simeq y'$	$\gamma\theta_y$
$\theta_z = \frac{v_z}{\beta c} \simeq \frac{\delta p}{p}$	θ_z

Stripping Rate Estimates

- IC 2 CW Lattice (3 variants)
- 3-8 GeV Pulsed Linac (short + long pulse variants)

- Tracking done using TraceWin/PARTRAN
- Use 2nd order moments and emittances computed by PARTRAN. (These moments are computed in the lab frame) and computes:
- Lattice functions from statistical definitions
- **Local velocity** spreads from $\sqrt{(\varepsilon/\beta)}$
- Intra Beam Stripping rate along the linac using formula

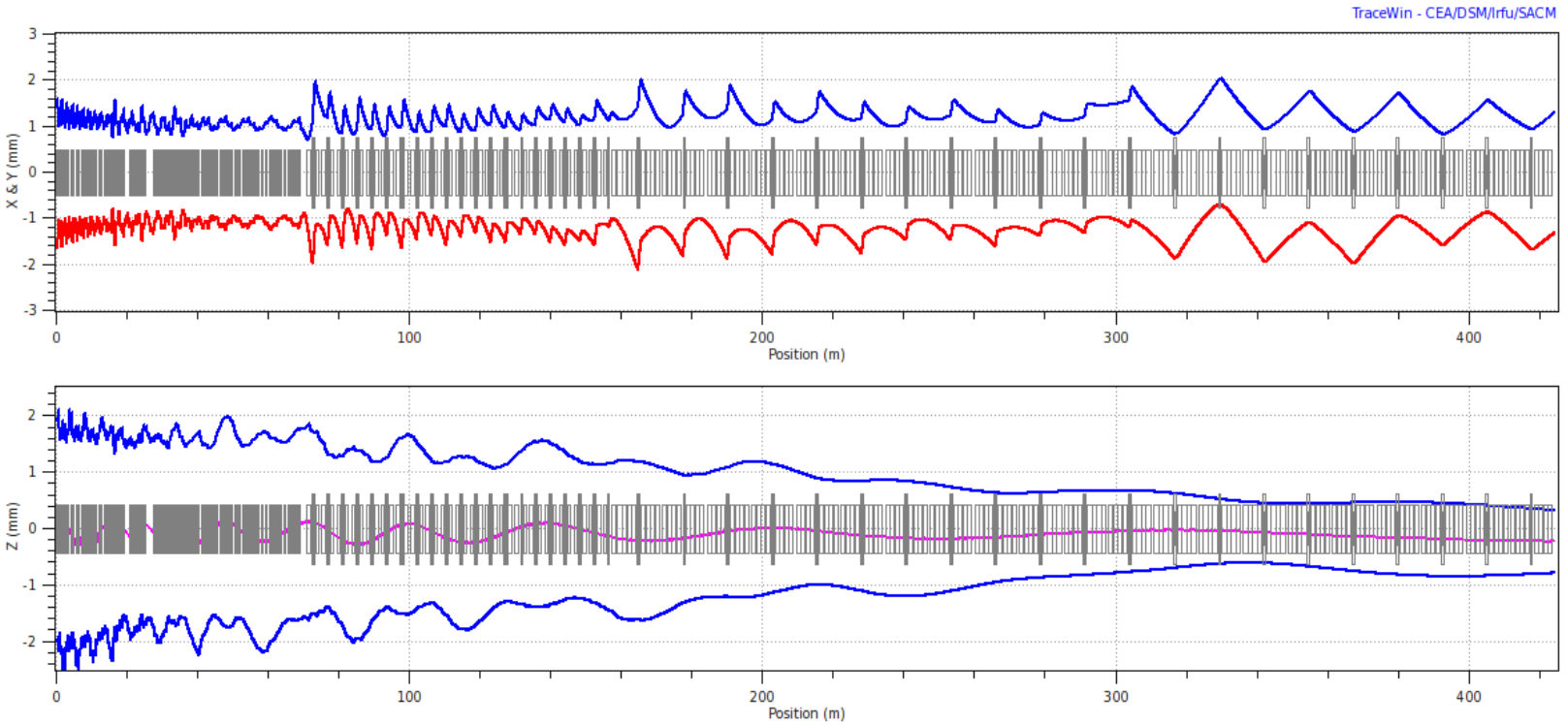
PARTRAN Output



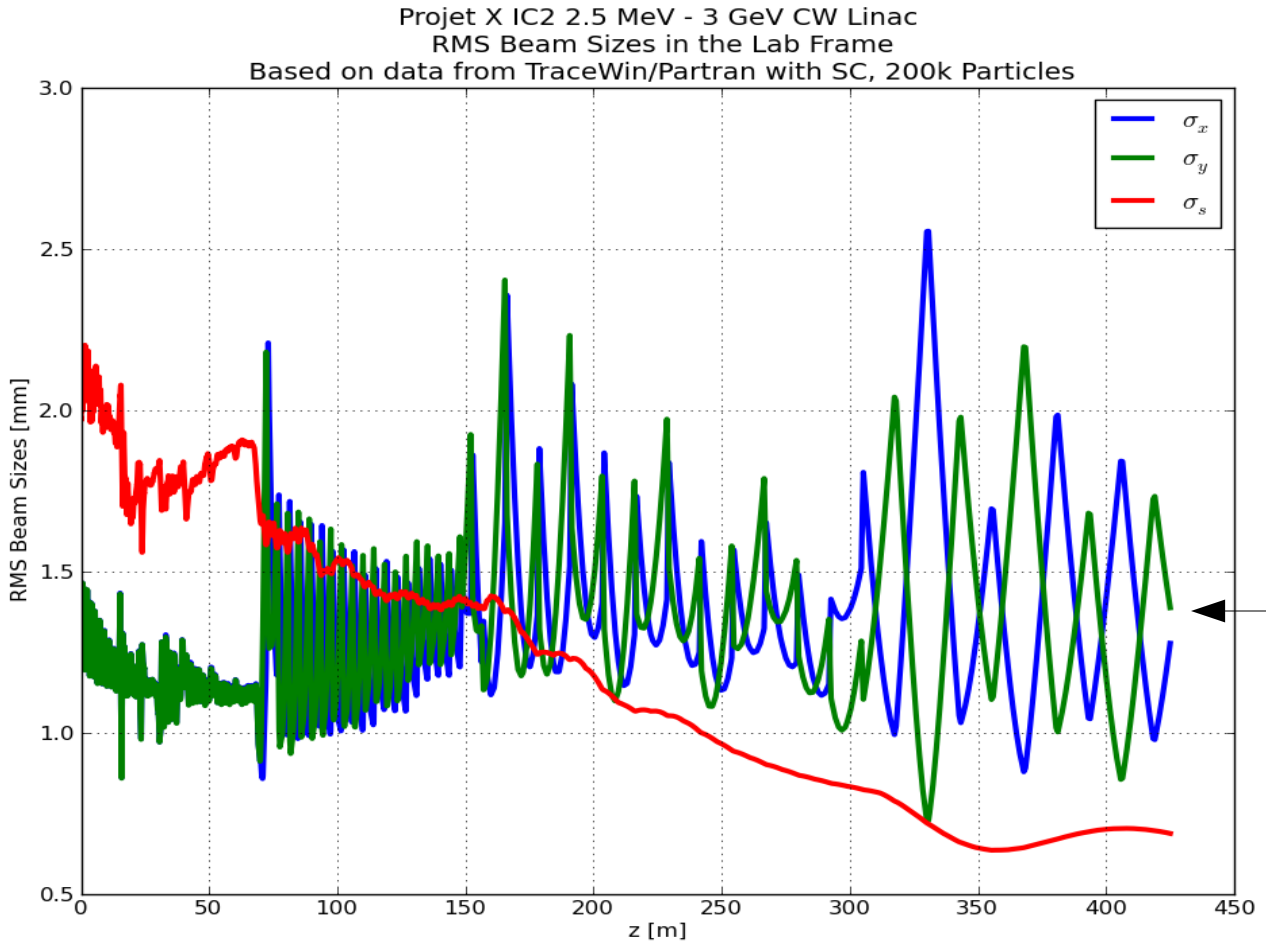
Multiple integration steps / element

PARTRAN provides emittances and 2nd order moments computed from the distribution.

IC2 CW Linac - Baseline



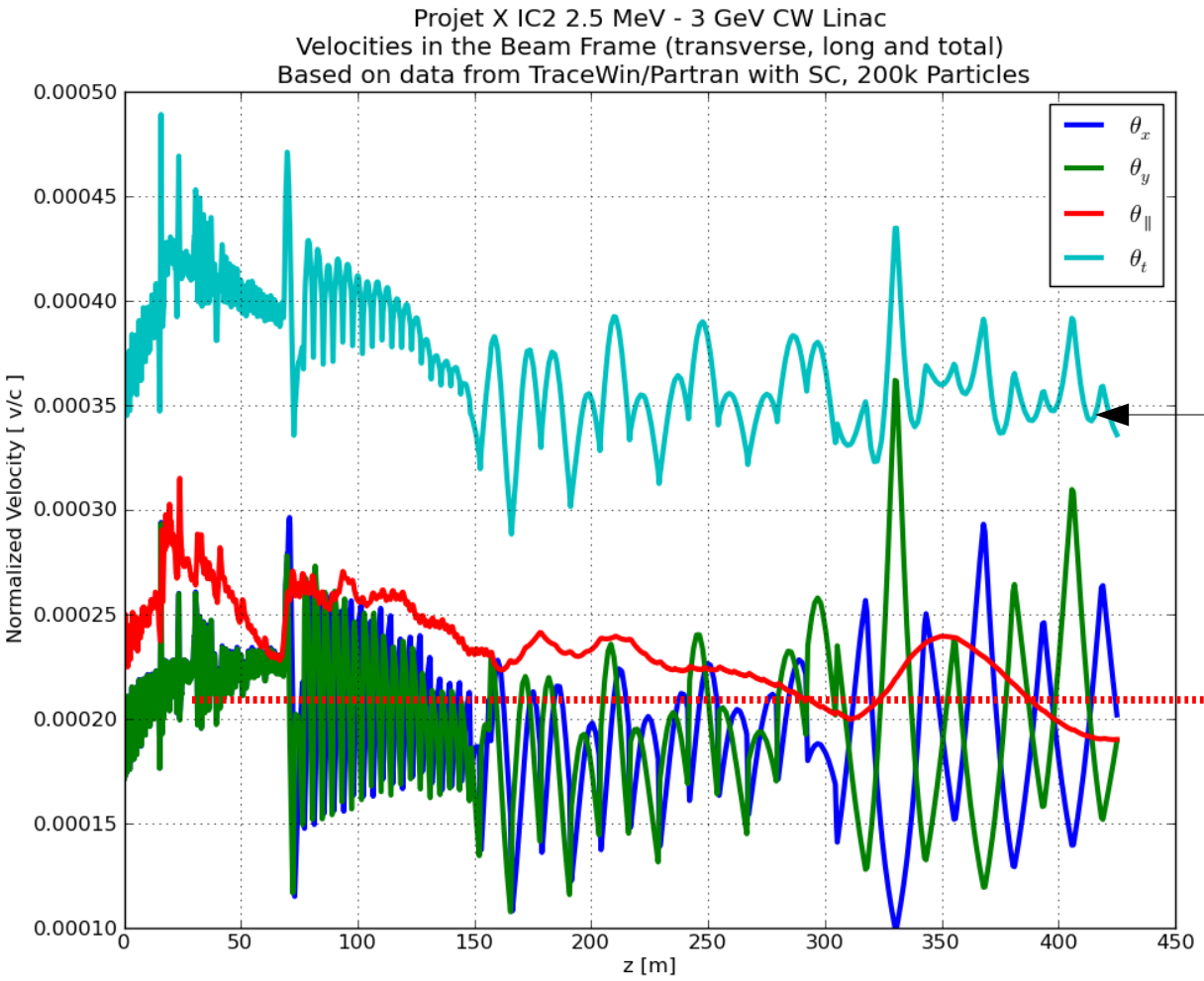
PrX- Beam Sizes In the Laboratory Frame



~ 1.5 mm

This is the same as what was shown on the earlier slide. Scales easier to read.

Velocity Spreads in the Beam Frame



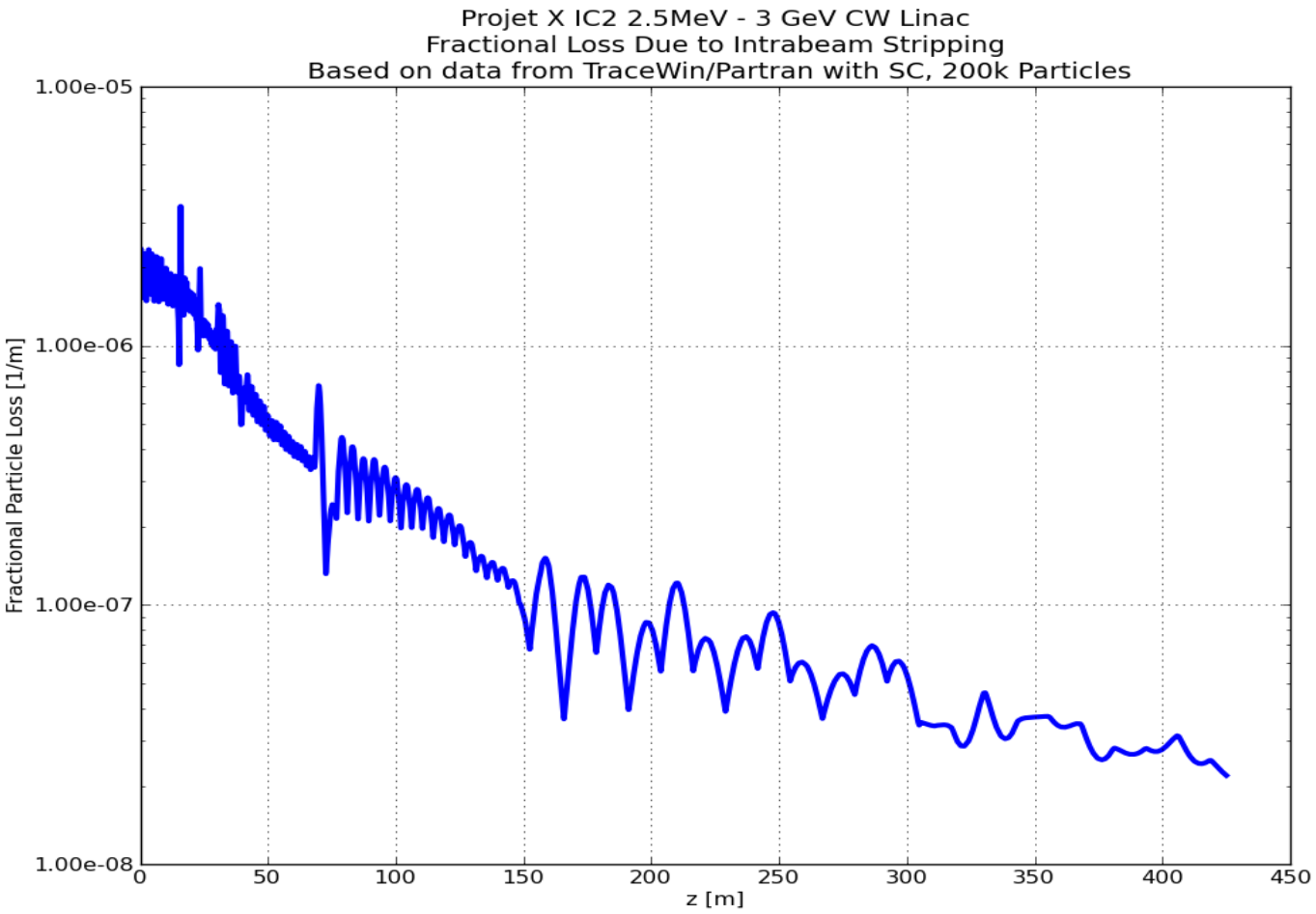
Velocity spread magnitude within constant cross-section approximation range of validity

$v/c \sim 2.0 \times 10^{-4}$

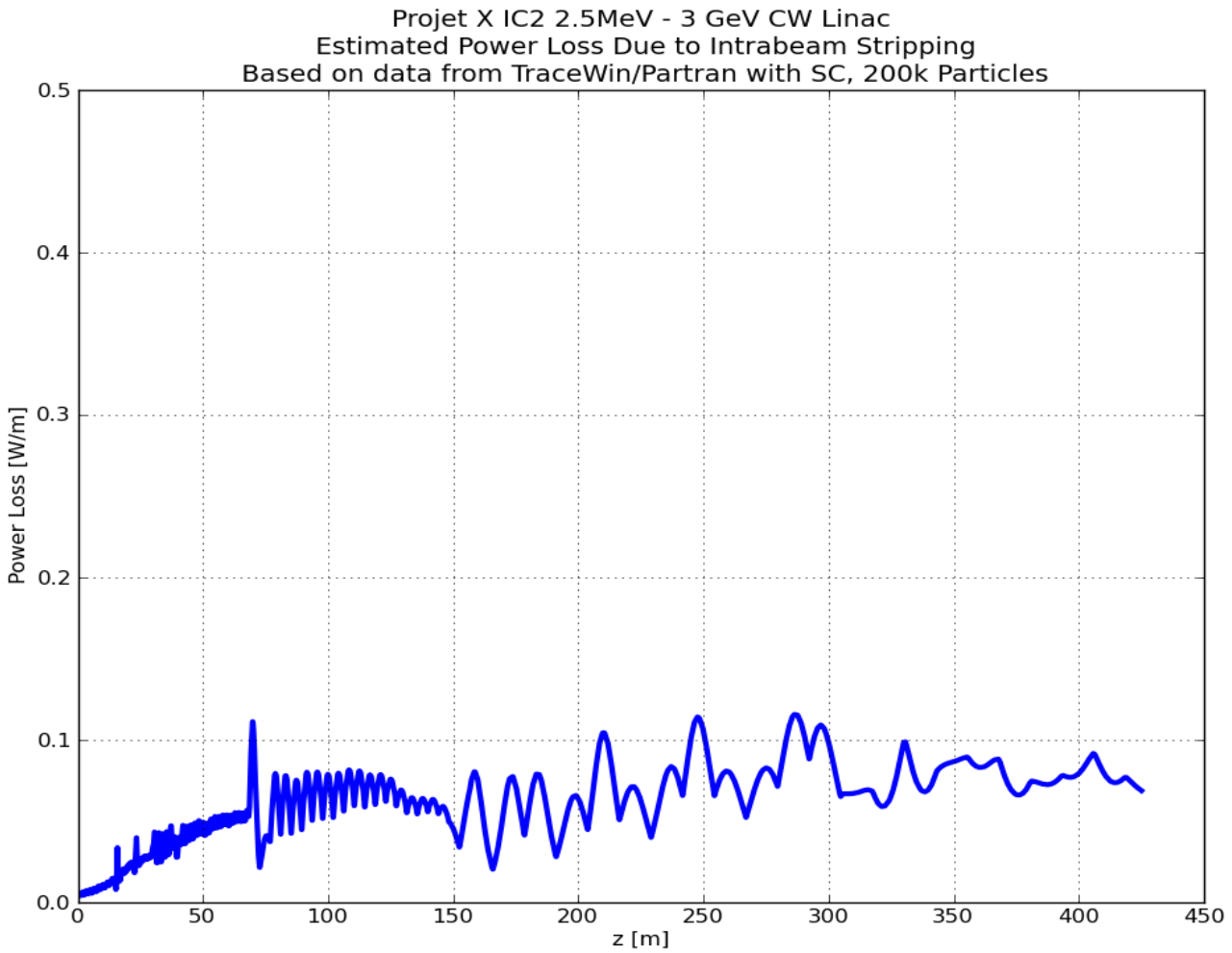
Lower bound of the constant cross-section approximation validity

Fractional Particle Loss

650 MHz + 1.3 GHz (2-3 GeV)



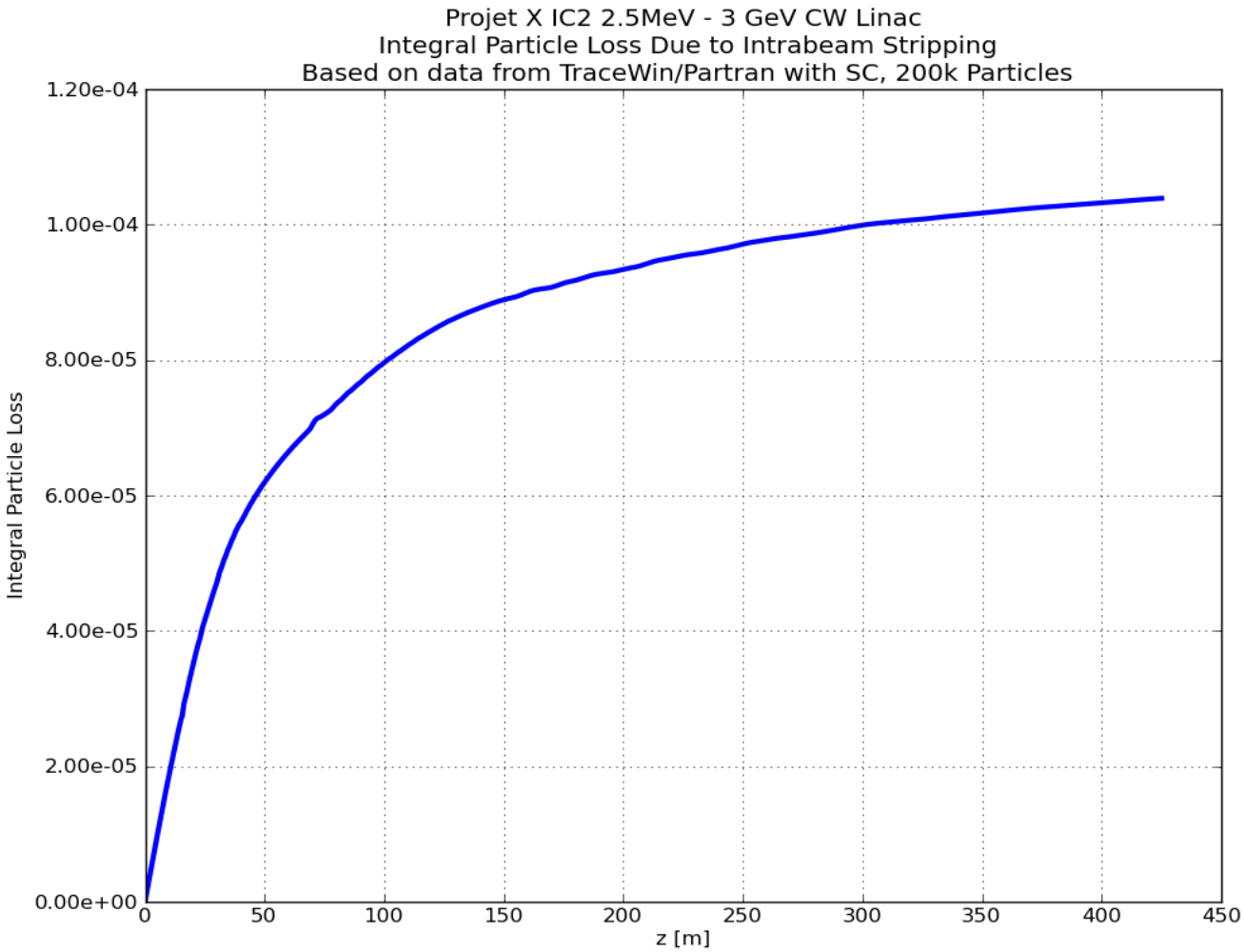
Avg Power Loss/unit length



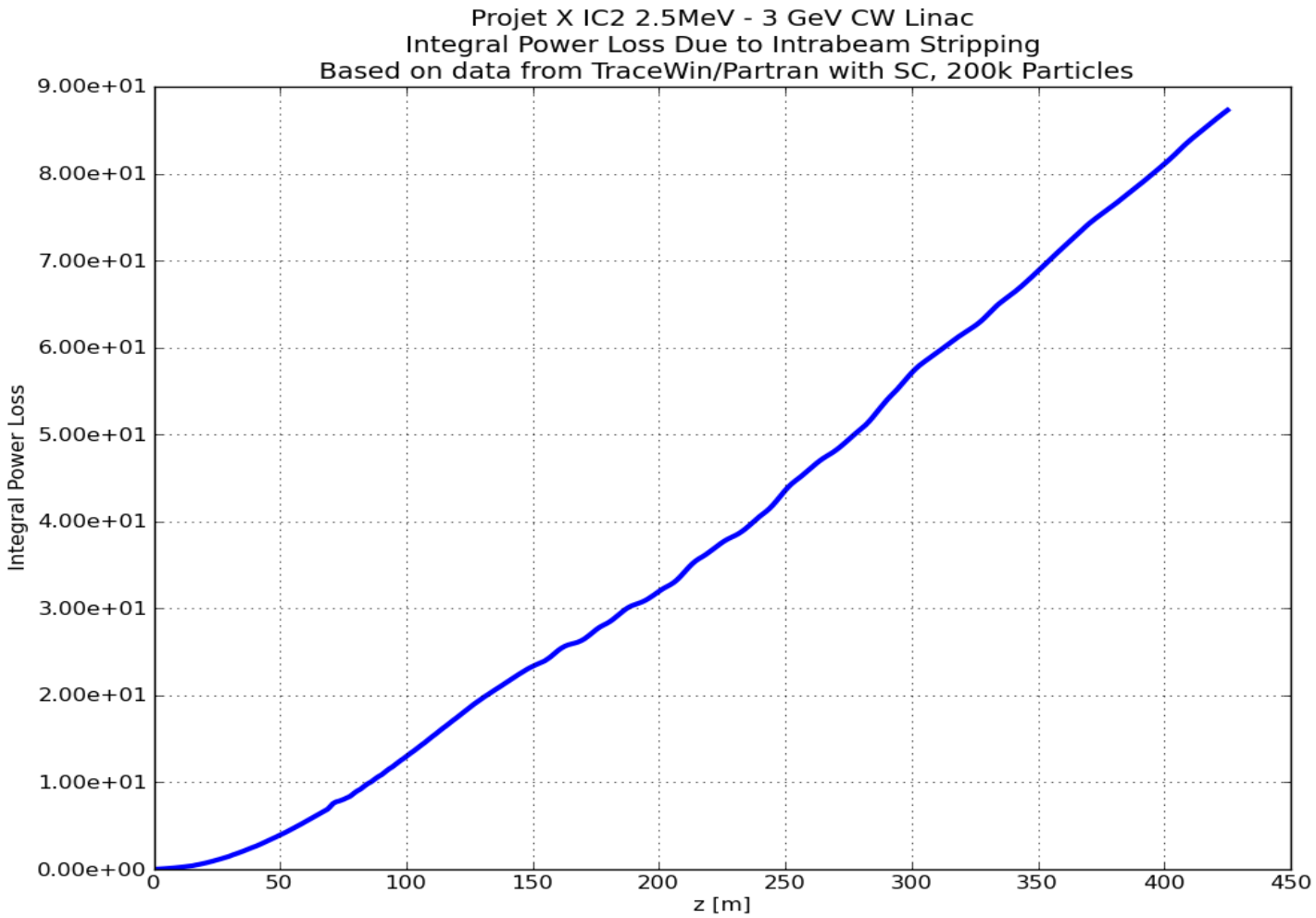
0.1 W/m

Average power loss assumes 1 mA average beam current

Integral Particle Loss

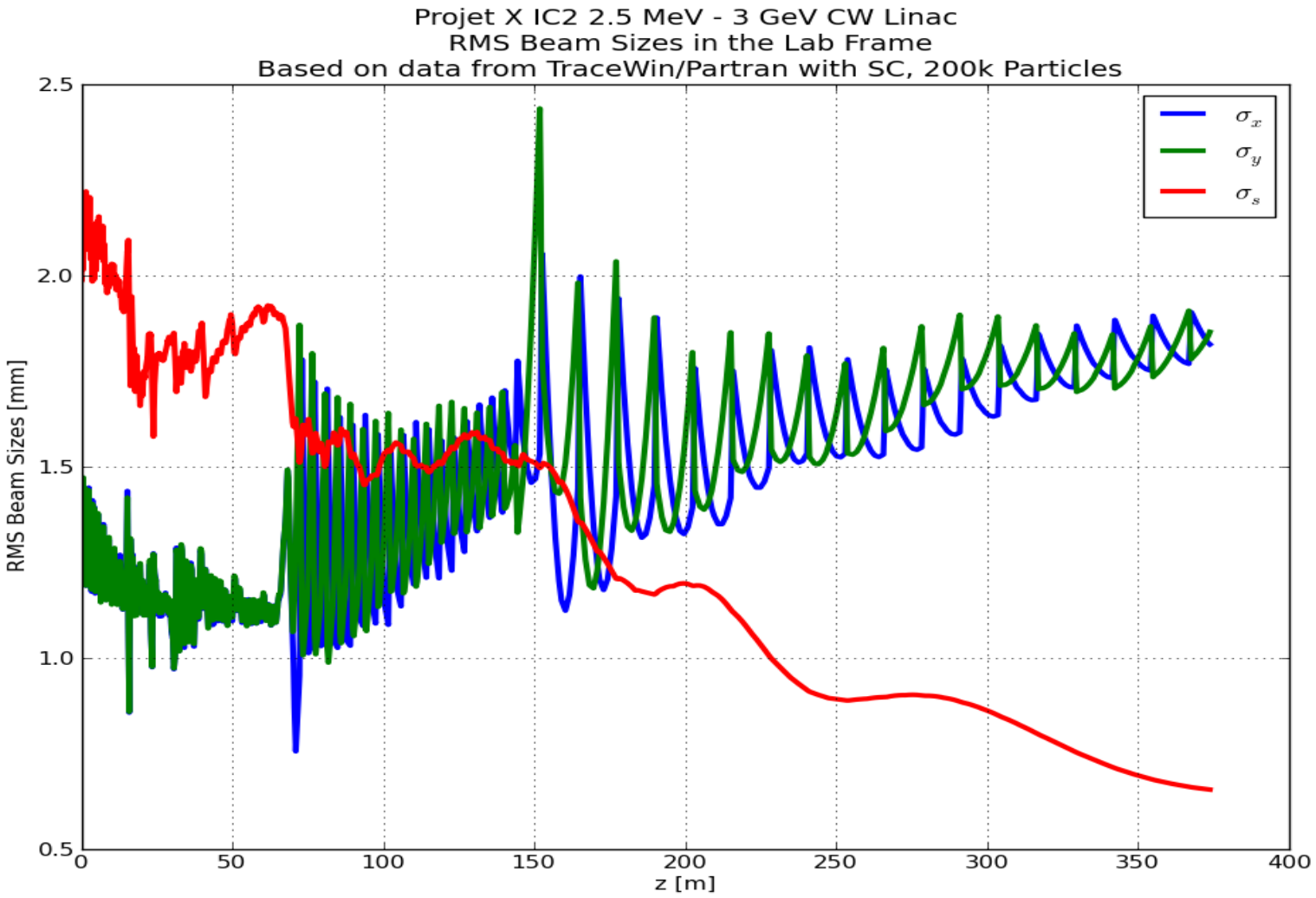


Integrated Power Loss

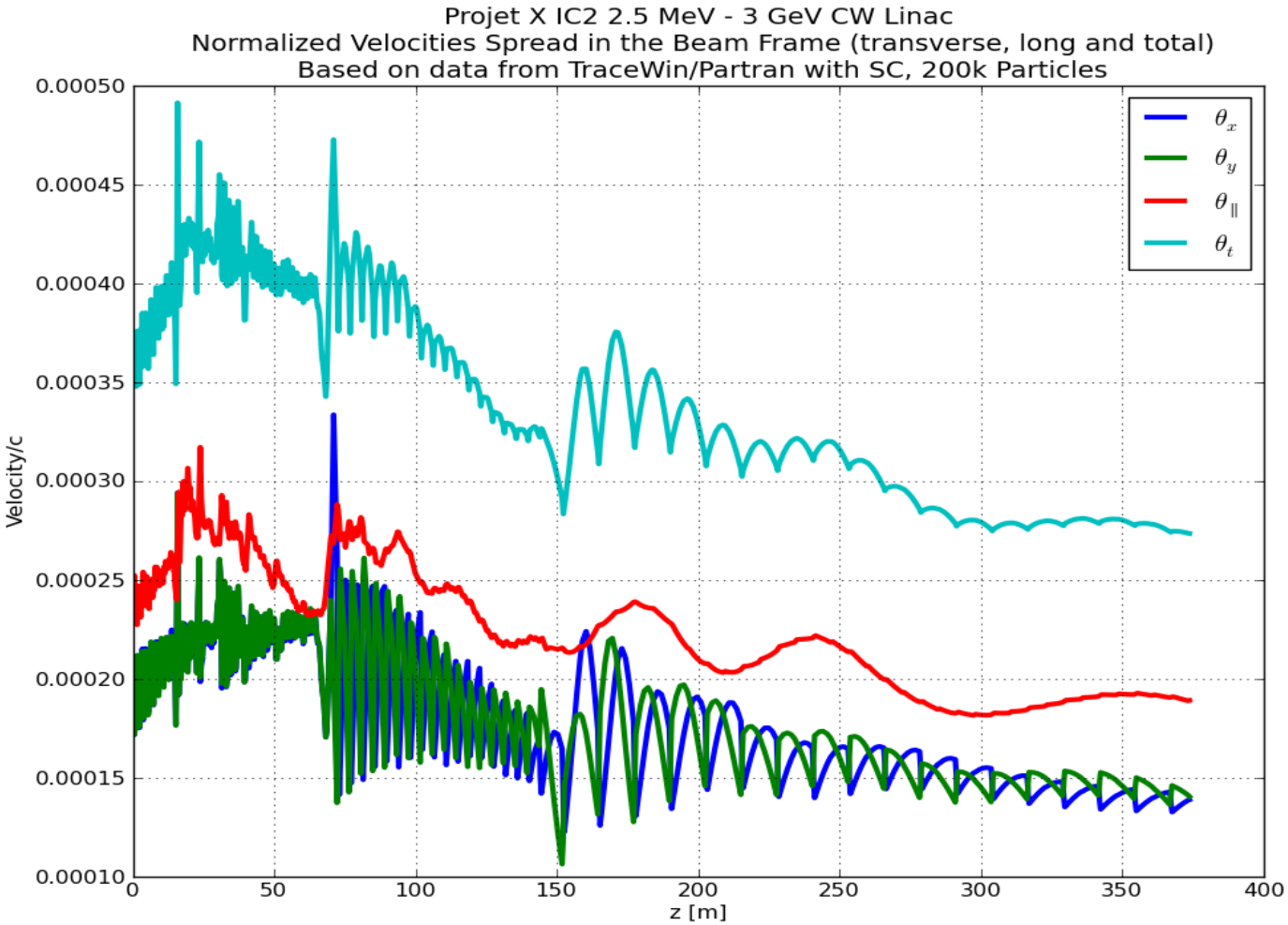


IBS Estimate in CW 650 MHz Variant no 1
650 MHz elliptical cavities 2-3 GeV
("rematched baseline")

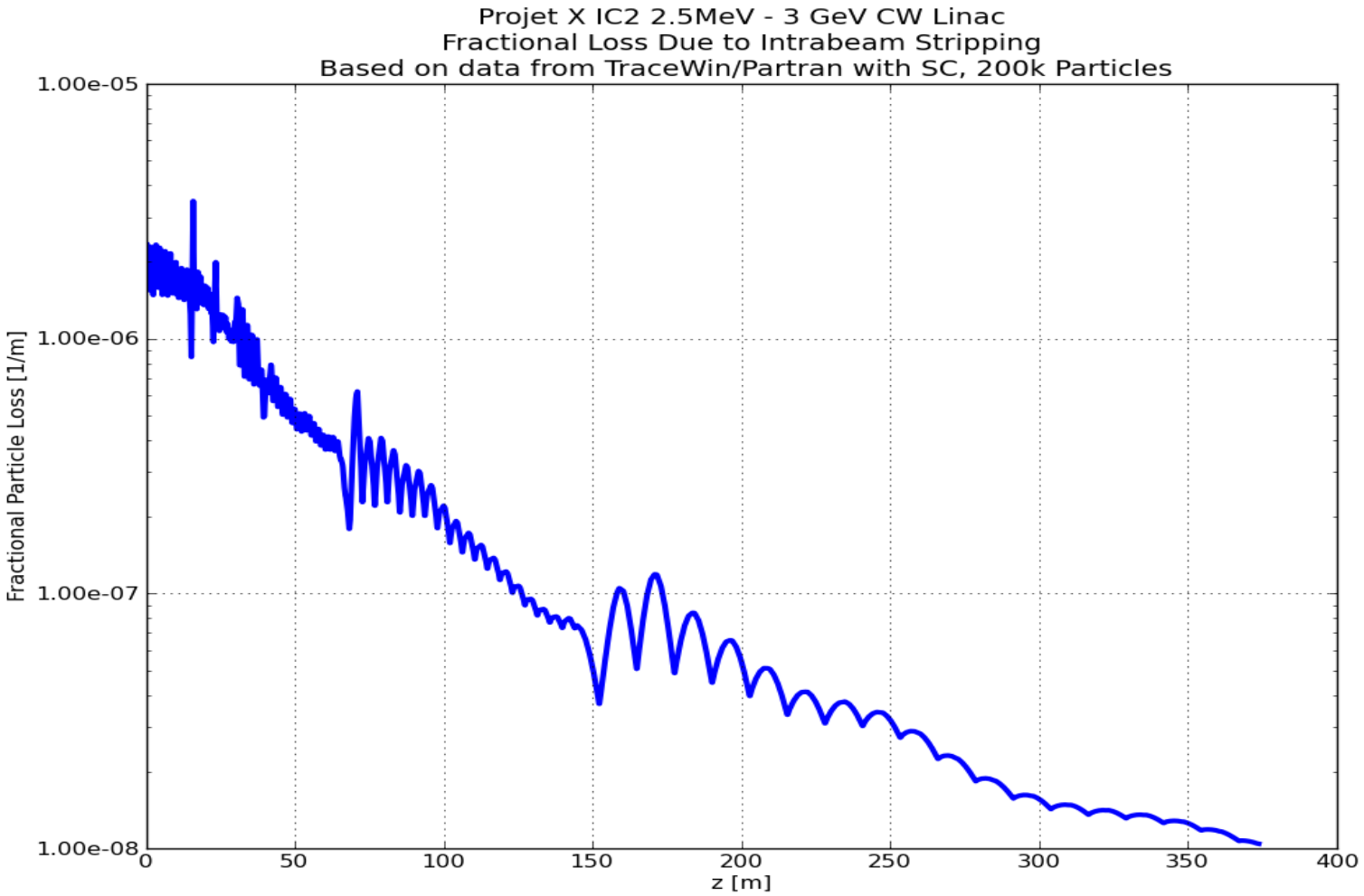
SC Linac V1 : Beam Sizes



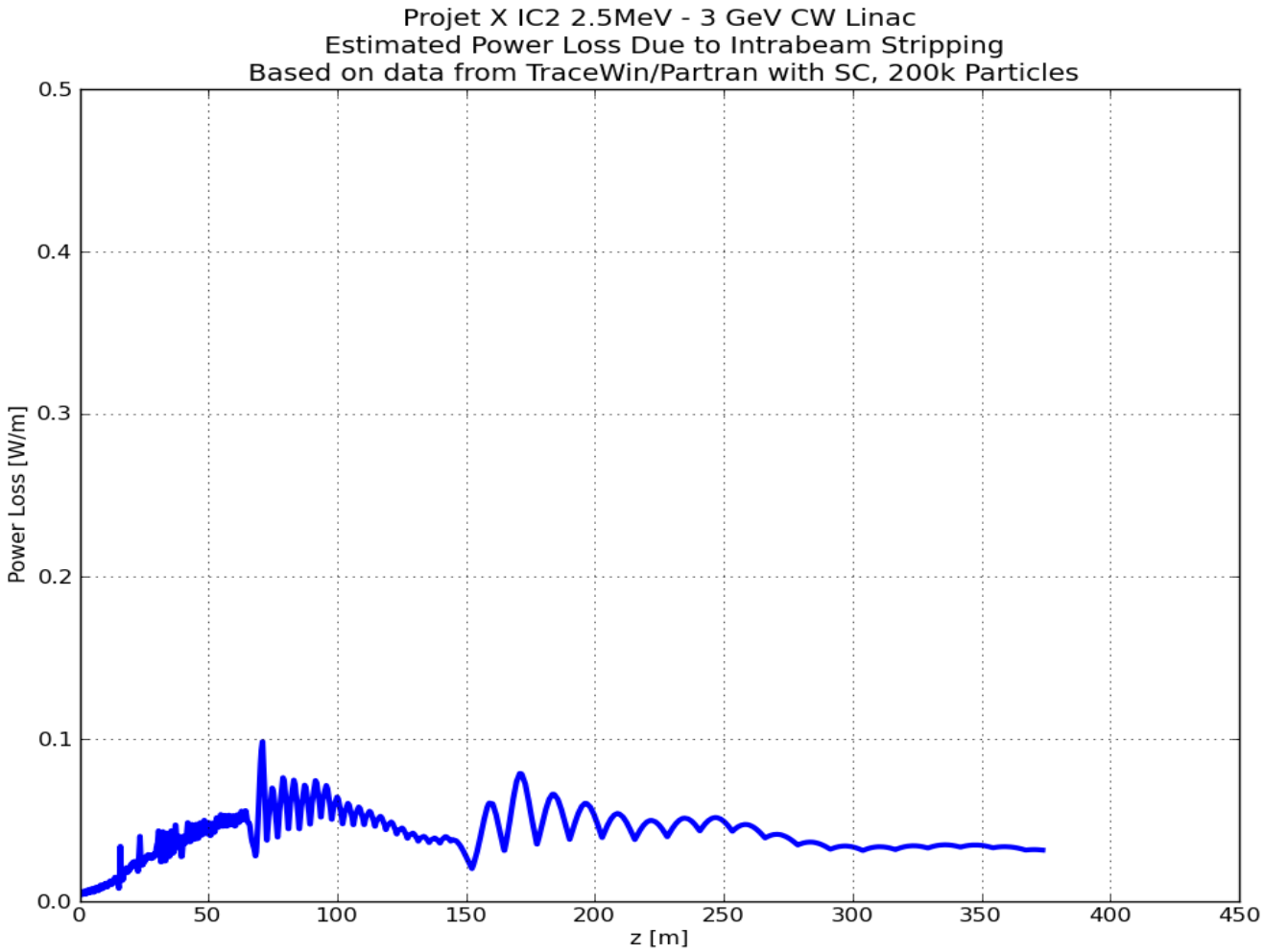
SC Linac V2: Velocities



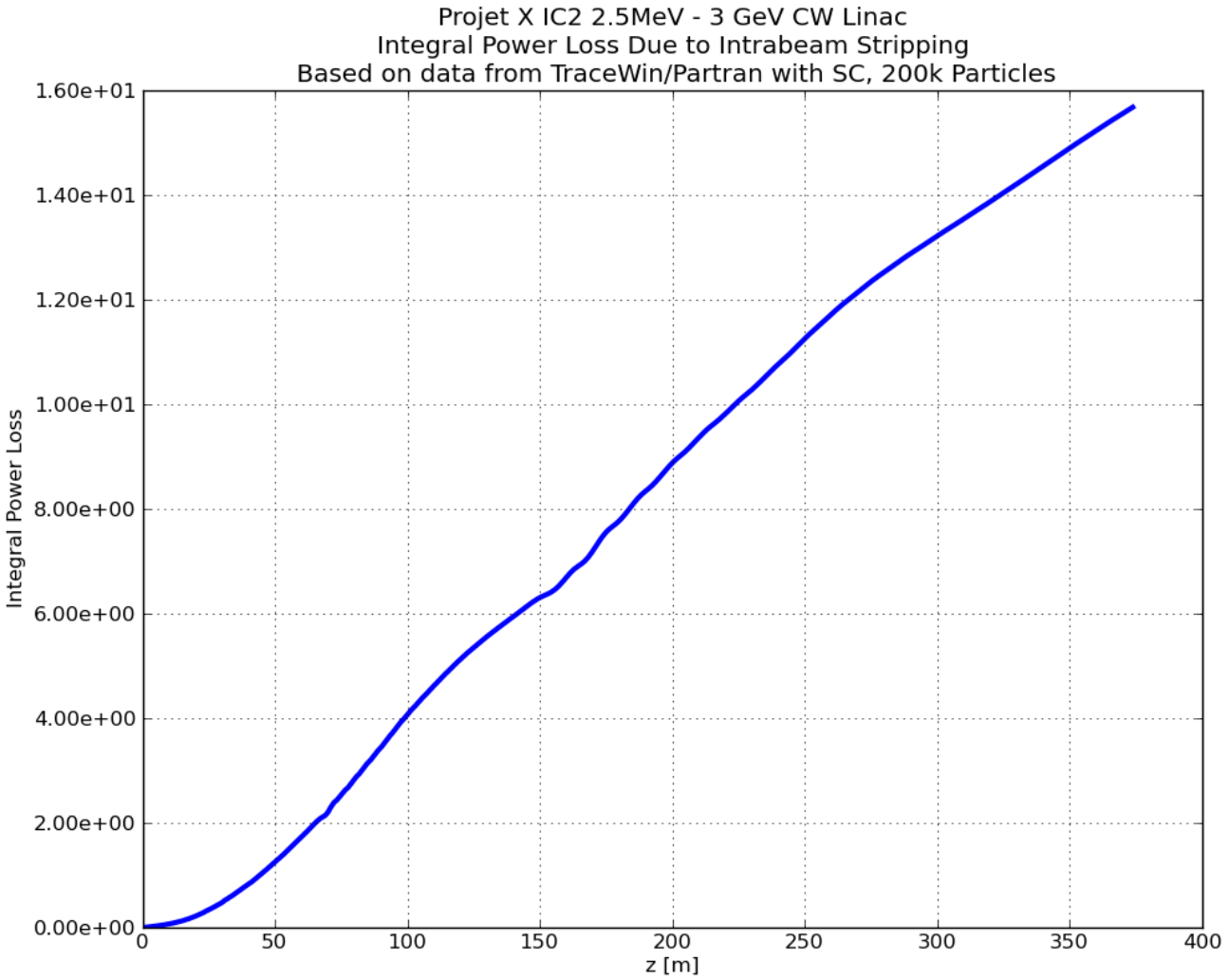
SC Linac V1: Fractional Loss



CW Linac V1: Avg Power Loss

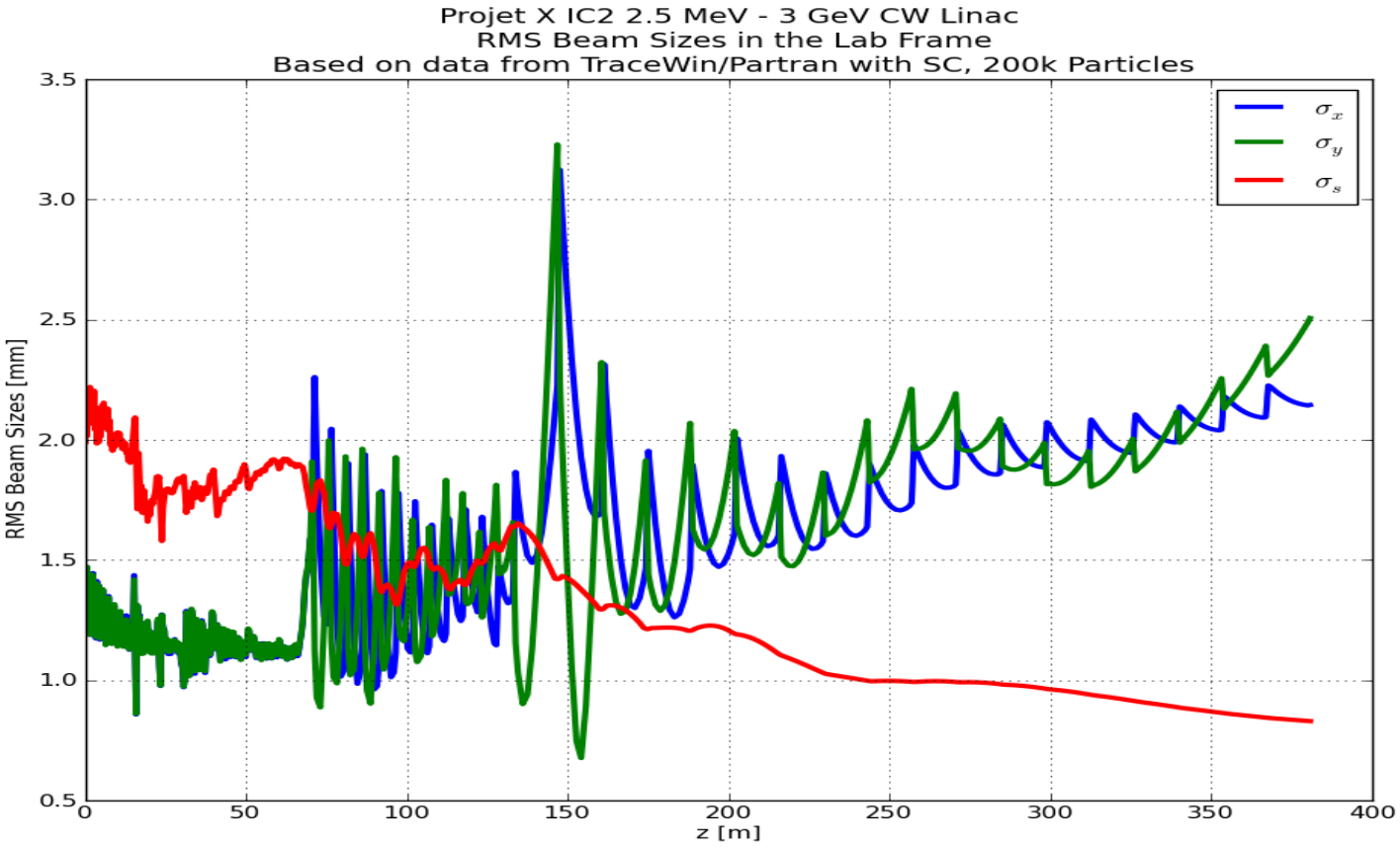


CW Linac V1 : Integrated Power Loss

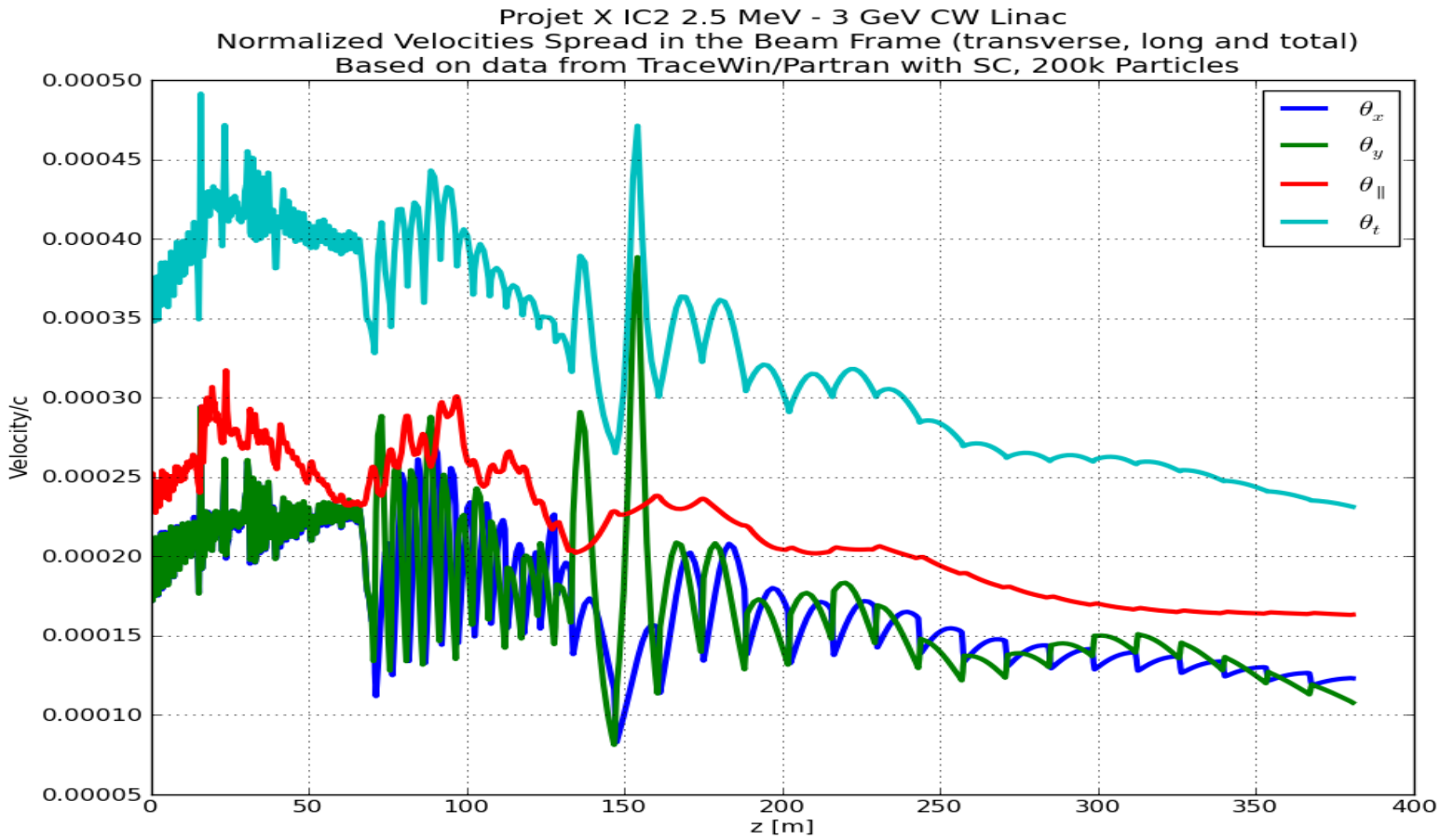


IBS Estimate in CW 650 MHz Variant no 2
(650 MHz 1-3 GeV, optimized with GenLinWin)

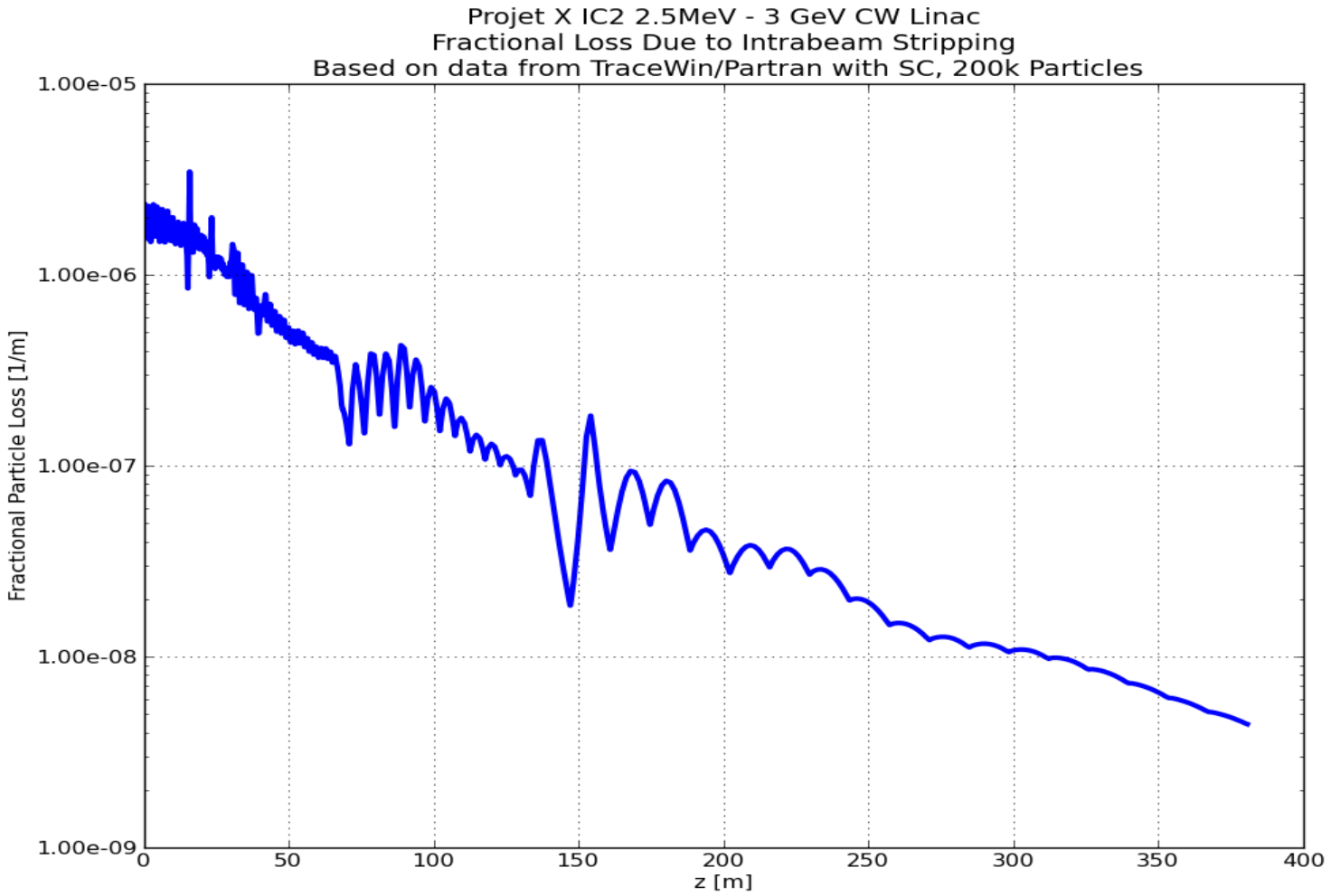
CW Linac V2: Beam Sizes



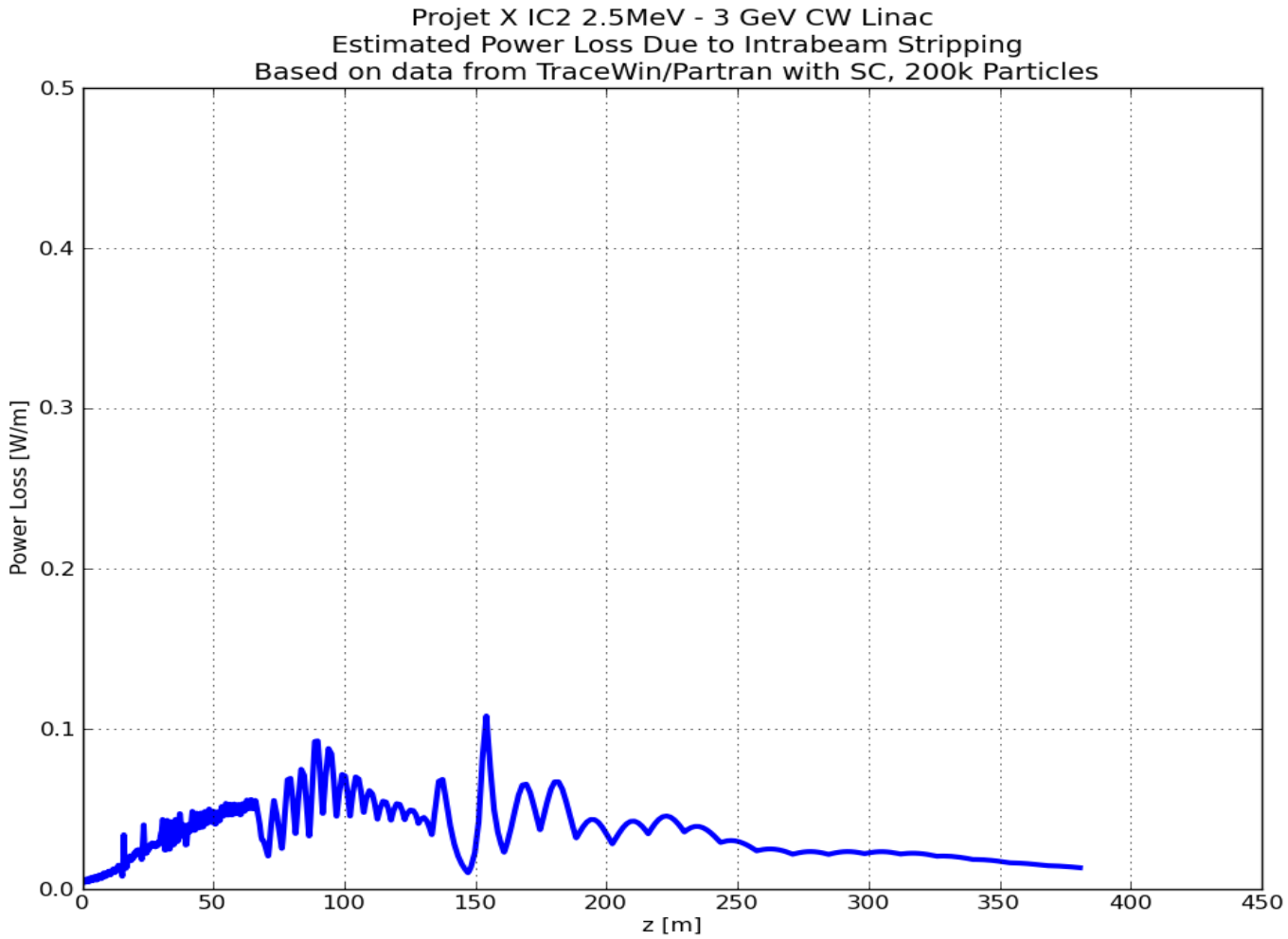
CW Linac V2: Velocities



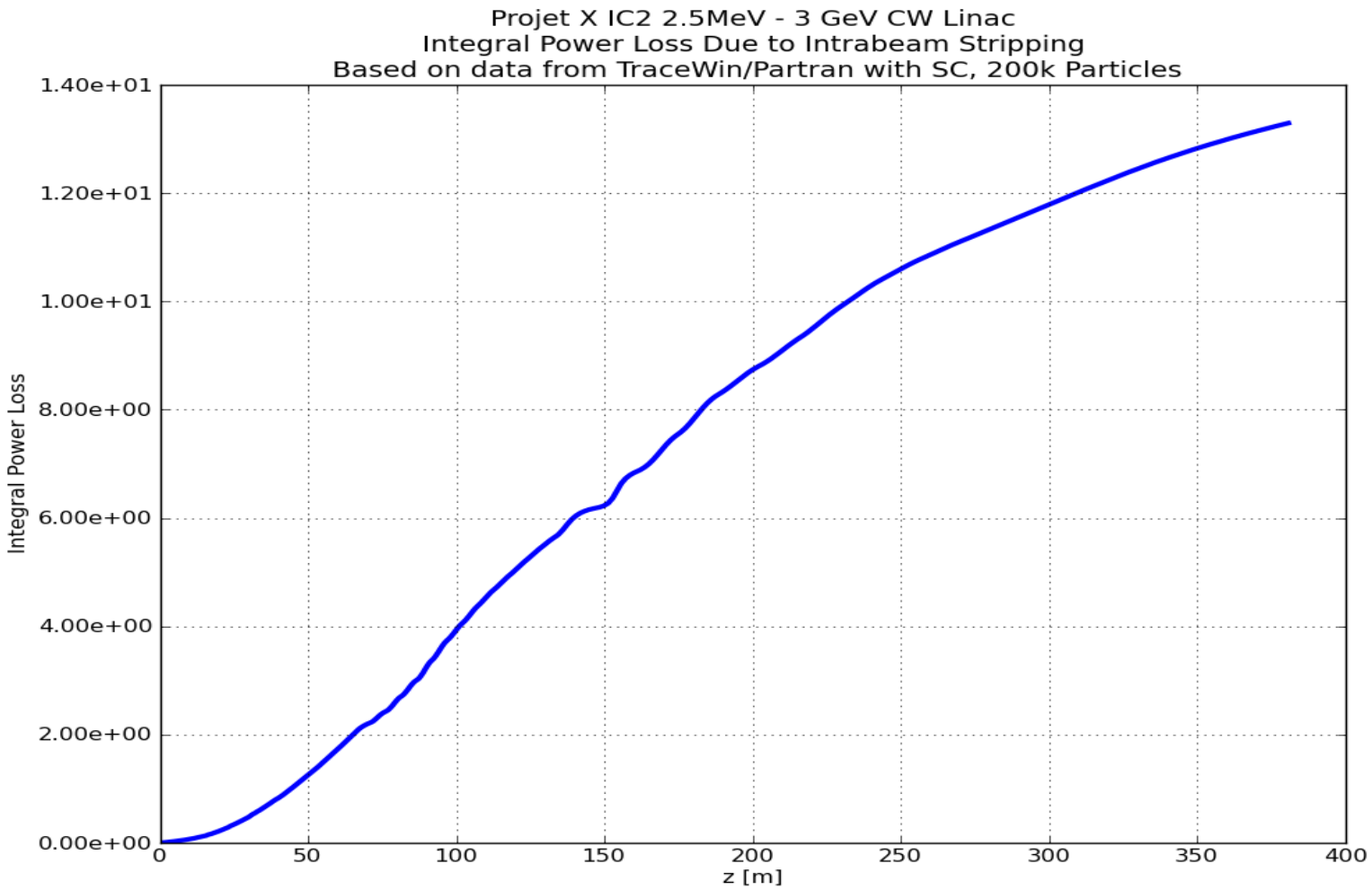
CW Linac V2: Fractional Loss



CW Linac V2: Avg Power Loss

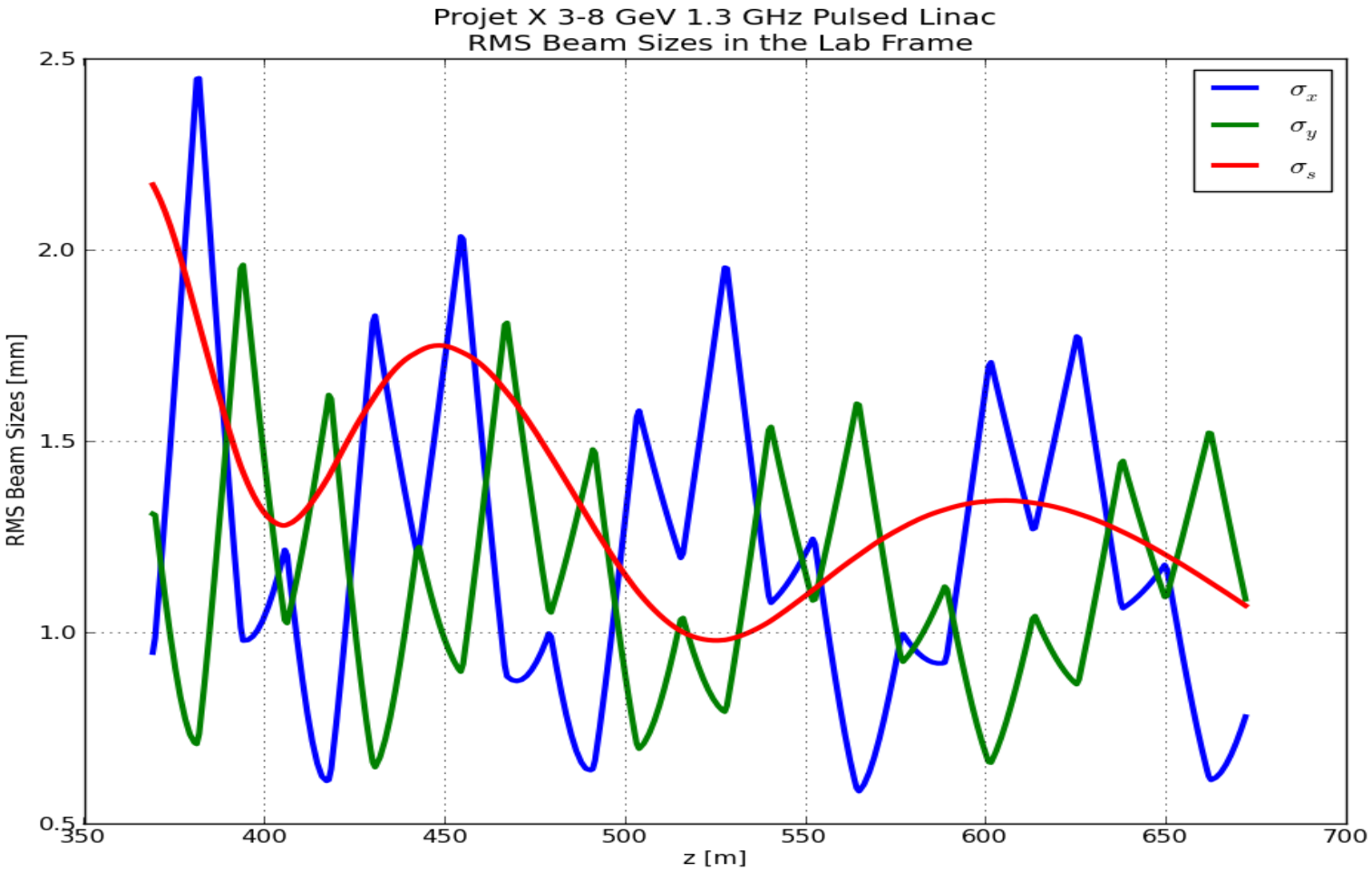


CW Linac V2: Integrated Power Loss

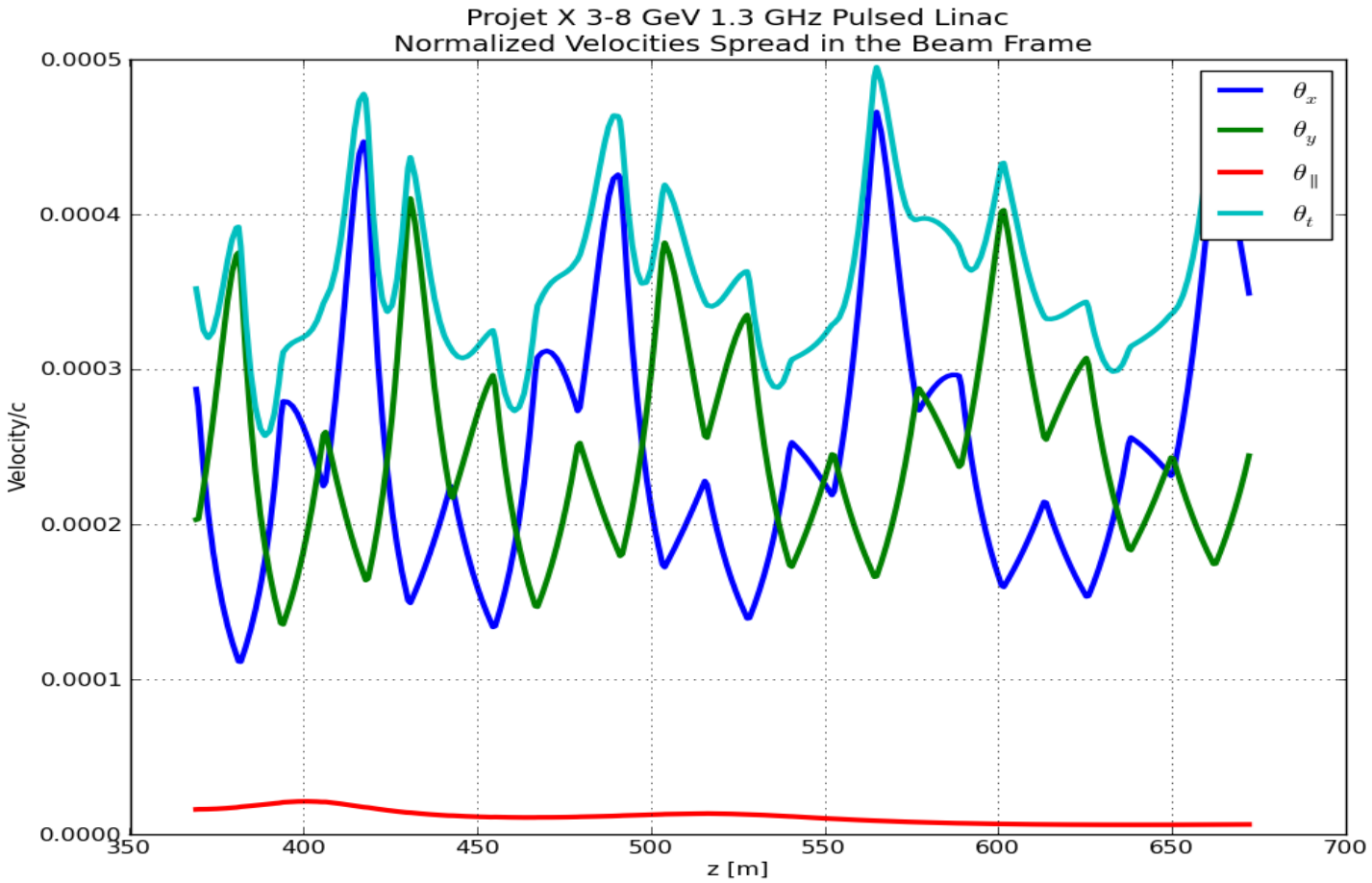


Pulsed Linac 3-8 GeV
1 mA peak , 0.5 duty factor

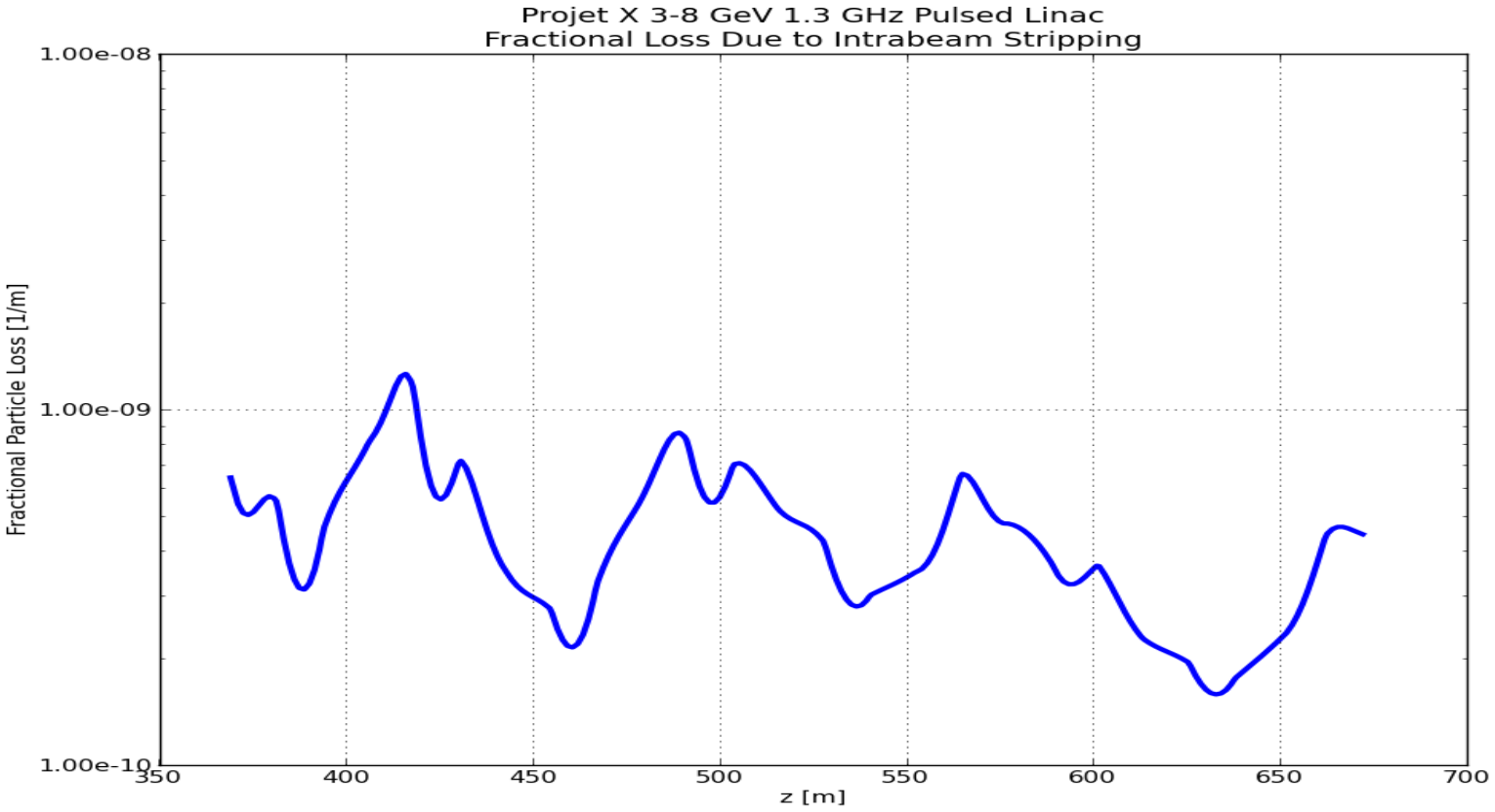
Beam Sizes


Projet X 3-8 GeV 1.3 GHz Pulsed Linac
RMS Beam Sizes in the Lab Frame
RMS Beam Sizes [mm]
z [m]
σ_x
σ_y
σ_s
2.5
2.0
1.5
1.0
0.5
350
400
450
500
550
600
650
700

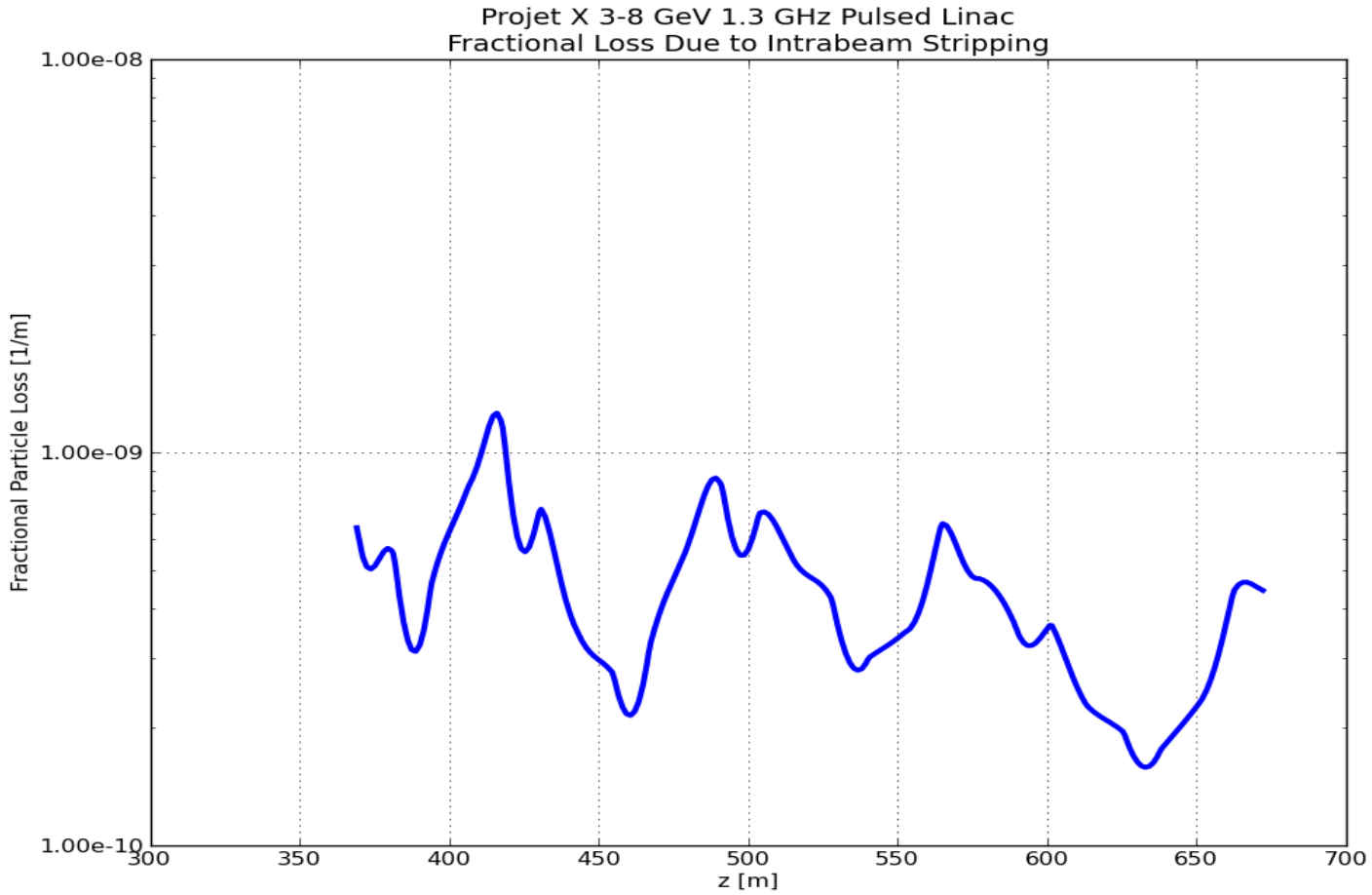
Velocities


Projet X 3-8 GeV 1.3 GHz Pulsed Linac
Normalized Velocities Spread in the Beam Frame
Velocity/c
z [m]
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
350
400
450
500
550
600
650
700
θ_x
θ_y
$\theta_\parallel$
θ_t

Fractional Loss (1 mA peak, DF=0.5)

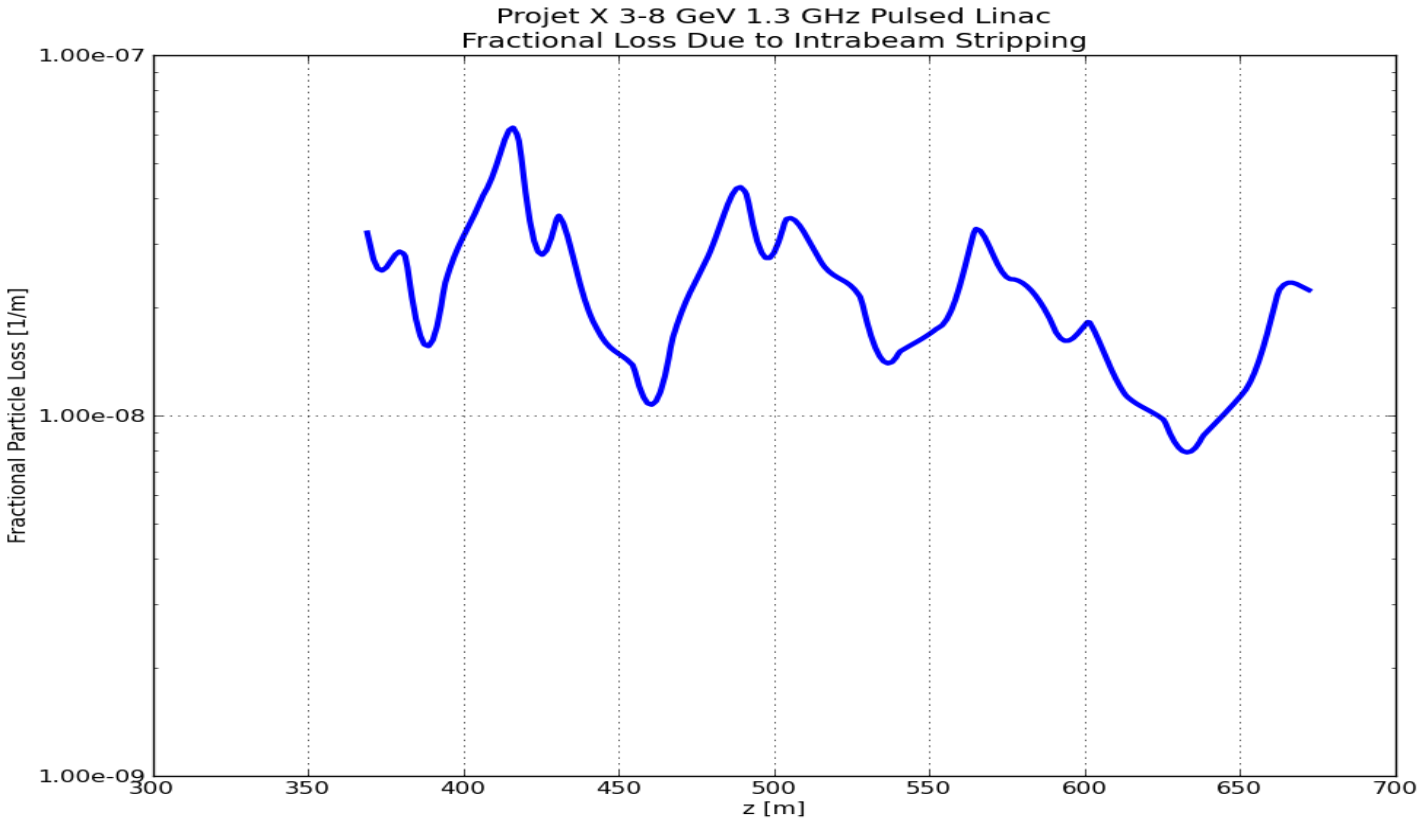


Power Loss (1 mA peak, DF = 0.5)

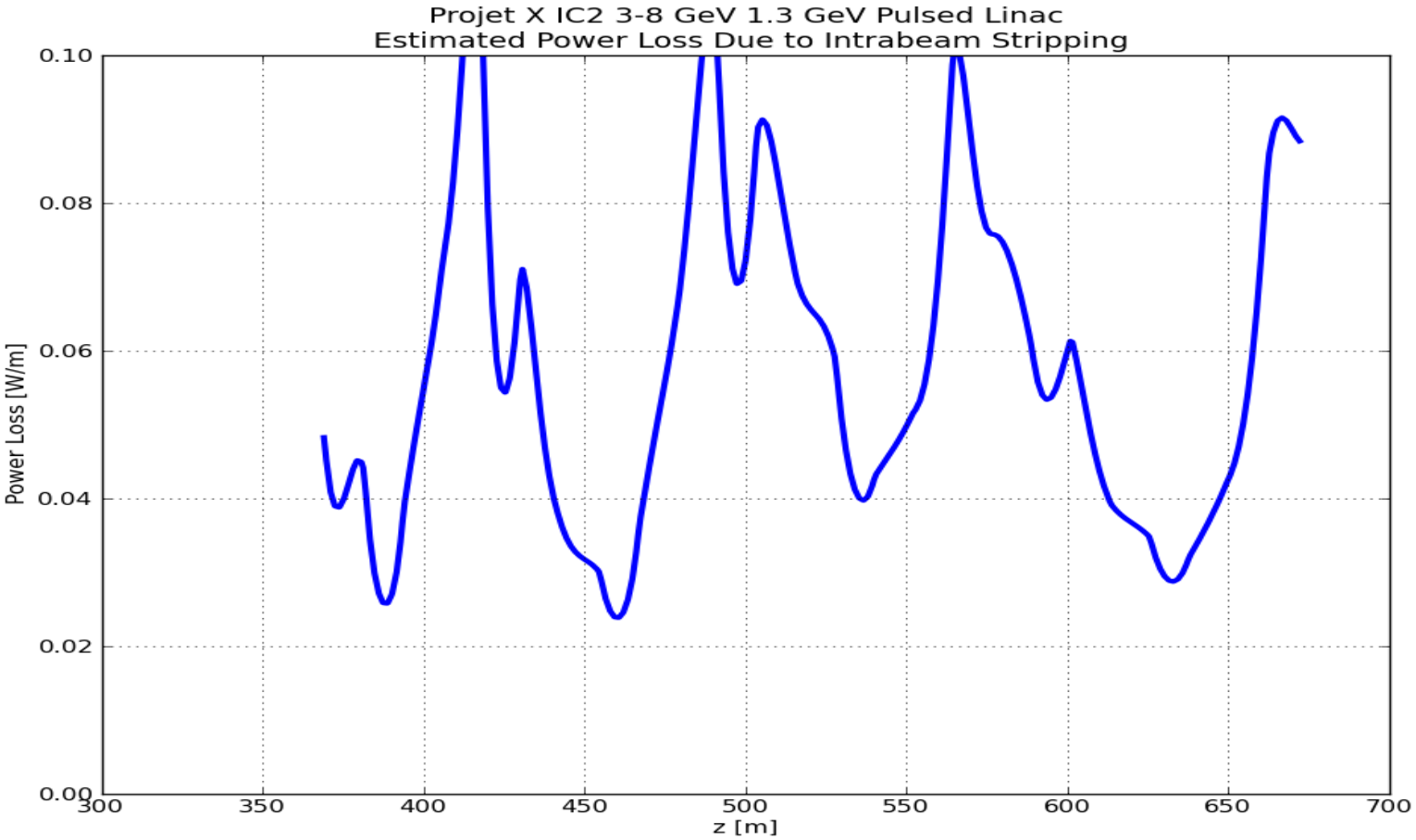


3- 8 GeV Pulsed Linac
50 mA, 0.01 Duty Factor

Project X Fractional Loss (50 mA peak, 0.01 DF)



Power Loss (50 mA, 0.01 DF)



Conclusions ?

- Assuming the IBS model is adequate, the predicted power loss caused by H- stripping appears tolerable in the current IC2 CW lattice.
- Further studies and experimental results from SNS will be helpful to gain more confidence about the validity of the model.
- 0.1 W/m is manageable but nevertheless remains a concern, especially in view of the uncertainties.
- As expected, the 650 MHz only lattice variants have somewhat lower losses (weaker focusing)…
- Pulsed linac 3-8 GeV seems not be a concern.