Evaluation of Tritium Generation in the Soil under the PXIE Beam Dump (Problem of the Groundwater Activation)

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Abstract. It is shown, that an estimated level of tritium production in the soil under the PXIE absorber yields significantly smaller value than the maximum acceptable value determined by Fermilab safety regulations.

The radiation shielding of PXIE beam dump is planned to be performed by a concrete box-like enclosure with following sizes 2.5x2.5x1.8 m^3 (Figure 1). It is placed on the 8'' thick concrete floor of PXIE enclosure and is separated from the soil by 6'' thick gravel layer.

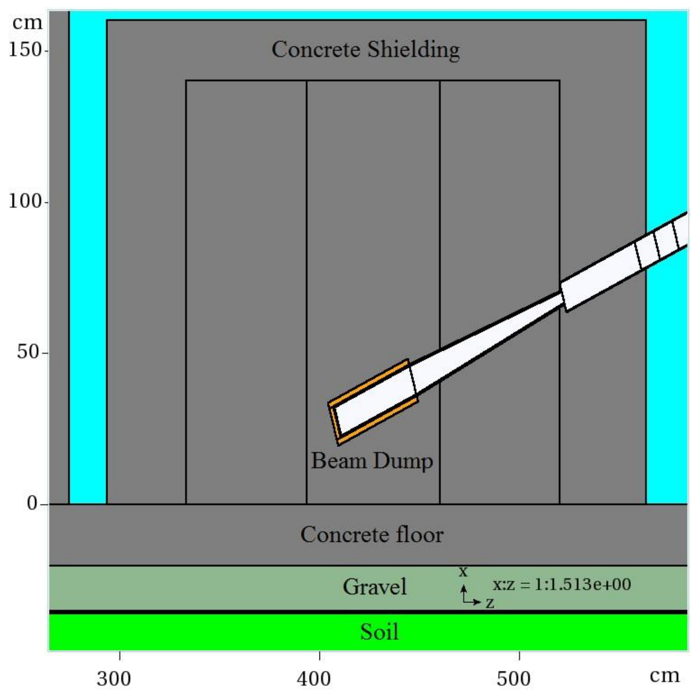

Figure 1: PXIE Beam Dump and its Concrete Shielding.

To evaluate the yield of tritium ($^3H_1$) in the soil under PXIE beam dump the following channels were considered:

- Production of radionuclides in materials of the absorber and its shielding with their subsequent migration into the soil;
- The production of tritium due to irradiation of the soil's lithium by neutrons produced in the beam dump.

The first channel, in our case is "closed" because the "stars" of nuclear reactions (they determines the production of radionuclides) are not formed at the beam energy below 30 MeV.

To evaluate the contribution of the second channel the following data were used:

- Composition of the soil (used in MARS): 57.152% - oxygen, 33.4417% - silicon, 7.14% - aluminum and 2.267% - hydrogen. It yields the molecular weight of the soil to be: $\mu_{soil} = 20.4857\ g/mol$;
- Density of the soil (used in MARS) – $\rho_{soil} = 2.25\ g \cdot cm^{-3}$;
- Abundance of lithium in the soil [1] – $\eta_{Li\_natural} = 29\ mg \cdot kg^{-1} = 2.9 \cdot 10^{-5}$, as it can be determined from Figure 2.
- Part of ${}^{6}\mathrm{Li}_3$ isotope in natural lithium [2] – $\eta_{{}^{6}Li_3} = 7.59\%$;
- Part of soluble lithium in natural lithium [3] – $\nu_{Li} = 5\%$.

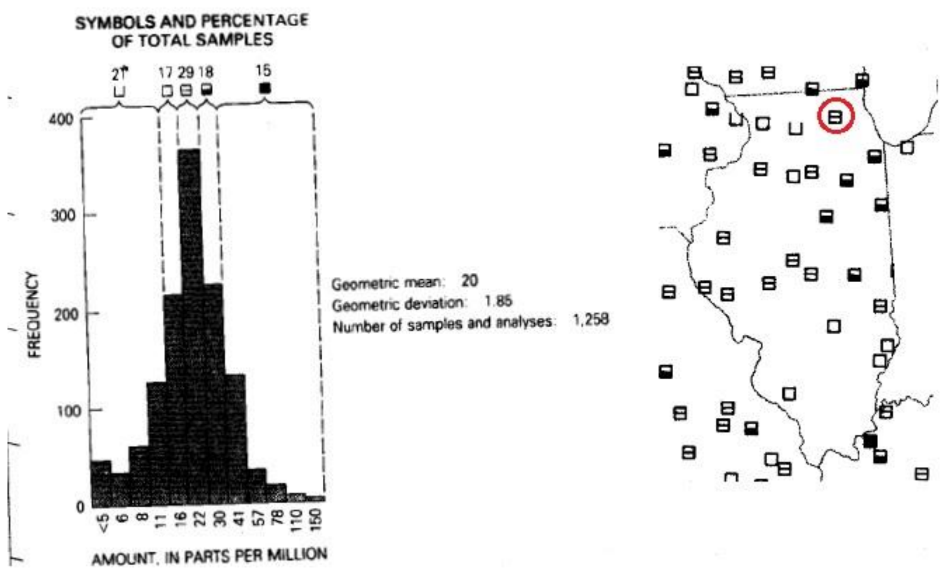

Figure 2: Abundance of lithium in the soil (state Illinois).

These data lead to the following value for concentration of ${}^{6}\mathrm{Li}_3$ atoms in the soil:

$$N_{{}^{6}Li_3} = 6.03 \cdot 10^{23} \cdot \frac{\rho_{soil}}{\mu_{soil}} \cdot \eta_{Li\_natural} \cdot \eta_{{}^{6}Li_3} \cdot \nu_{Li} = 7.45 \cdot 10^{15}\ \mathrm{cm}^{-3}.$$

To evaluate the rate of the tritium production due to nuclear reaction ${}^{6}\mathrm{Li}_3(n,\mathrm{A})^{3}\mathrm{H}_1$ the cross-section of this reaction must be known. These data cannot be found in the database EXFOR [4]. However the data can be determined from for the ratio of cross-sections of the required reaction to a "reference" reaction ${}^{10}\mathrm{B}_5(n,\mathrm{A})^{7}\mathrm{Li}_3$ and the cross-section of the "reference" reaction. Corresponding data are shown in Figures 3 and 4. The result of the linear interpolation of the data for cross-section of the "reference" reaction in the range from 10^{-8} MeV to 10^{-1} MeV is shown in Figure 5. It results in a power-law dependence with $\alpha_{CS} = -0.4940$ which for practical applications can be approximated by square root; so that in the above energy range the dependence of the cross-section on the neutron velocity is

$$\alpha_{CS} \propto 1/\mathrm{v}_n \propto 1/\sqrt{E_{kin}}\ .$$

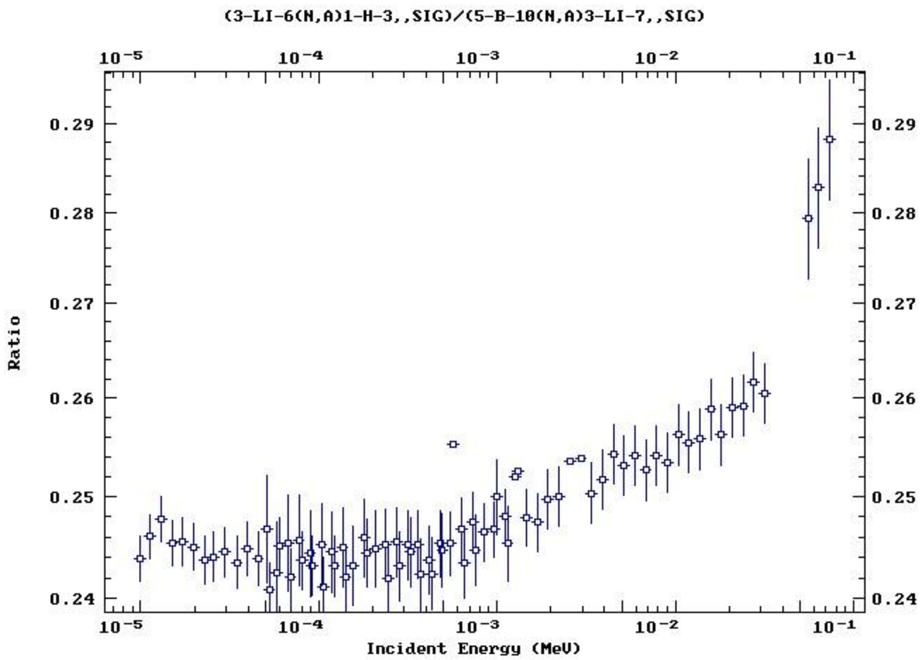

Figure 3: The ratio of cross-sections of the reactions ${}^{6}\mathrm{Li}_3(n,\mathrm{A})^{3}\mathrm{H}_1$ and ${}^{10}\mathrm{B}_5(n,\mathrm{A})^{7}\mathrm{Li}_3$.

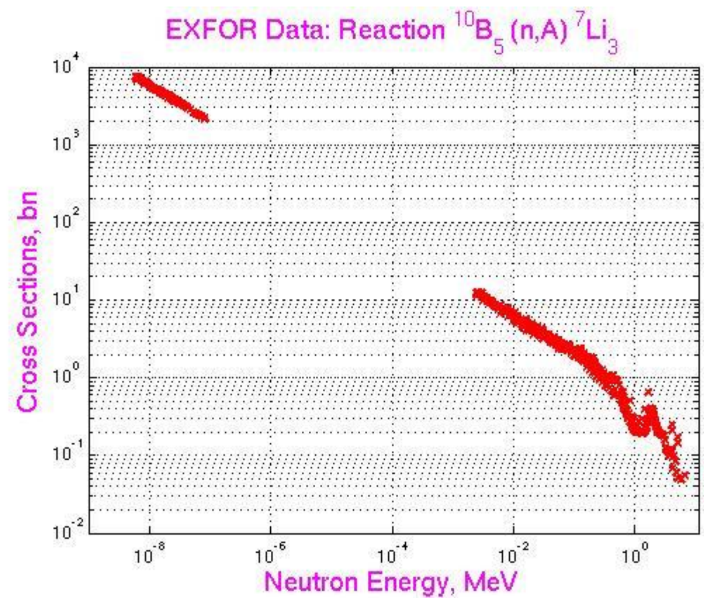

Figure 4: The cross-sections of the reaction ${}^{10}\mathrm{B}_5(n,\mathrm{A})^{7}\mathrm{Li}_3$.

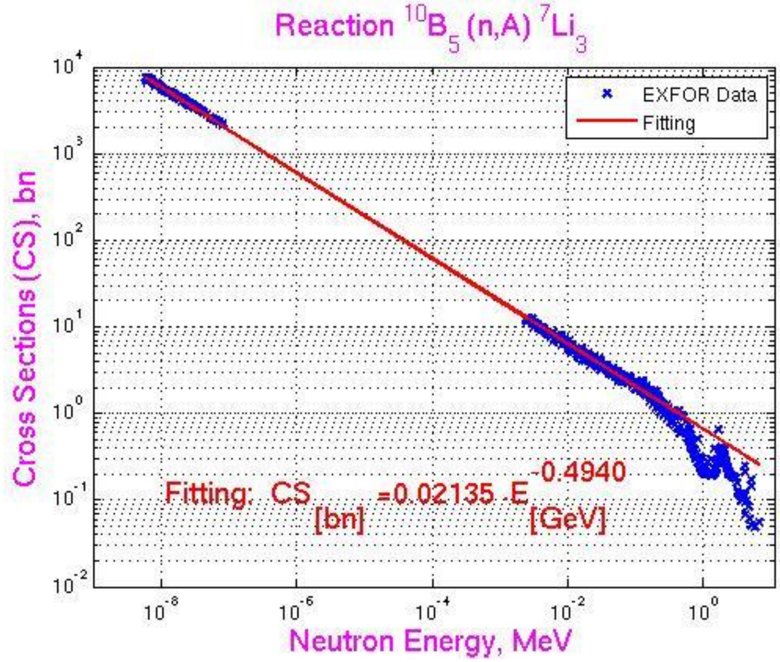

Figure 5: Linear interpolation of the data from Figure 4.

Figure 6 presents the results of combining data of Figures 3 and 5 presenting the dependence of the cross-section of the reaction $^{6}\mathrm{Li}_3(n,\mathrm{A})^3\mathrm{H}_1$ on the neutron energy (the cross-section of the "reference" reaction $^{10}\mathrm{B}_5(n,\mathrm{A})^7\mathrm{Li}_3$ is also shown).

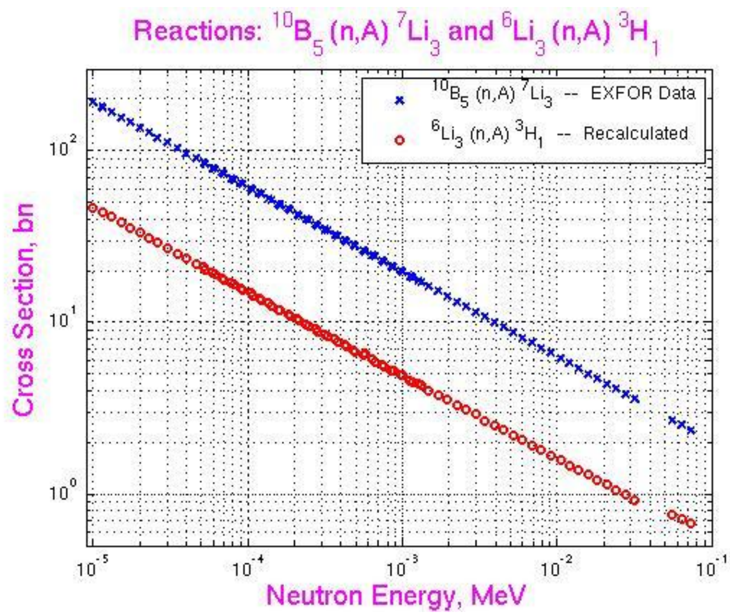

Figure 6: Cross-section of the reactions $^{6}\mathrm{Li}_3(n,\mathrm{A})^3\mathrm{H}_1$ and $^{10}\mathrm{B}_5(n,\mathrm{A})^7\mathrm{Li}_3$.

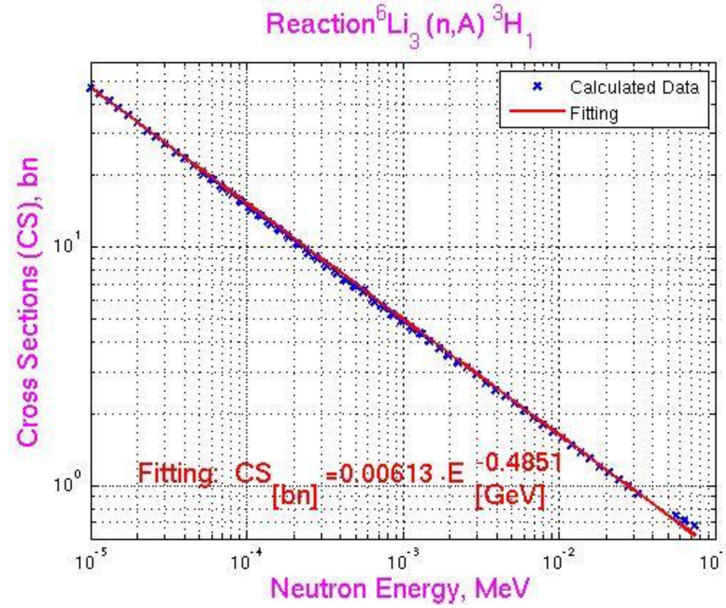

Figure 7: Cross-section of the reaction $^{6}\mathrm{Li}_3(n,\mathrm{A})^3\mathrm{H}_1$.

Figure 7 presents the result of linear interpolation of the data from Figure 6. One can see that

$$\sigma_{Li} = A_{Li} \cdot E_{[GeV]}^{-\alpha_{Li}}, \text{ where } A_{Li} = 0.00613 \text{ nb and } \alpha_{Li} = 0.4851.$$

To obtain an accurate estimate of tritium production in the case of strong dependence of cross-section on the energy one needs to know the neutron distribution on the energy. The corresponding spectrum obtained in MARS simulations is shown in Figures 8,9.

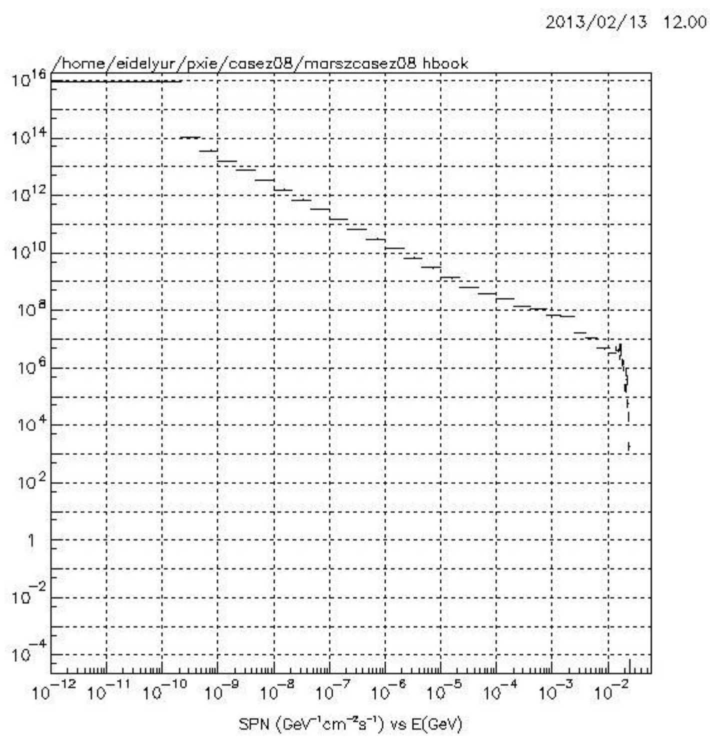

Figure 8: Spectral neutron distribution (soil directly under the PXIE beam dump).

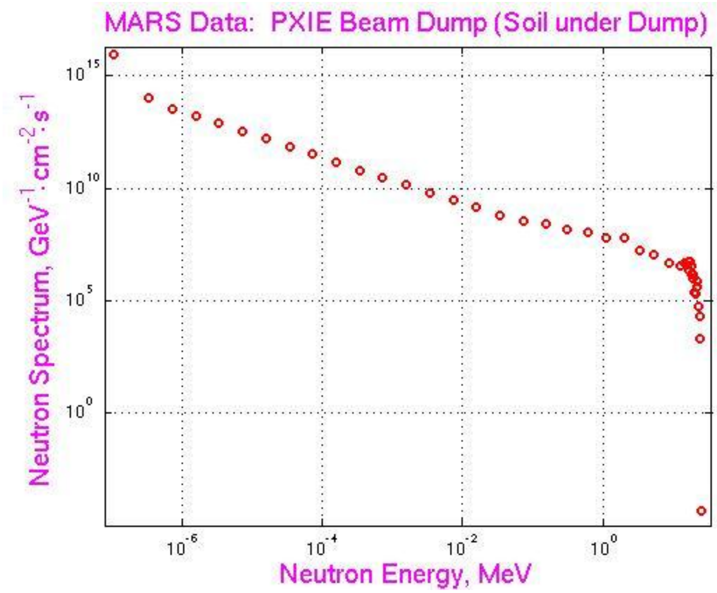

Figure 9: Spectral neutron distribution (soil directly under the PXIE beam dump; Area of interest).

The data on the cross-section of the reaction of tritium in the soil (Figures 6,7) exist only in the neutron energy range from $E_{\min} = 10^{-5} MeV$ to $E_{\max} = 10^{-1} MeV$. As one can see in Figure 8 in this energy range the neutron spectral distribution presented in the double logarithmic scale can be linearly interpolated with good accuracy as presented in Figure 10.

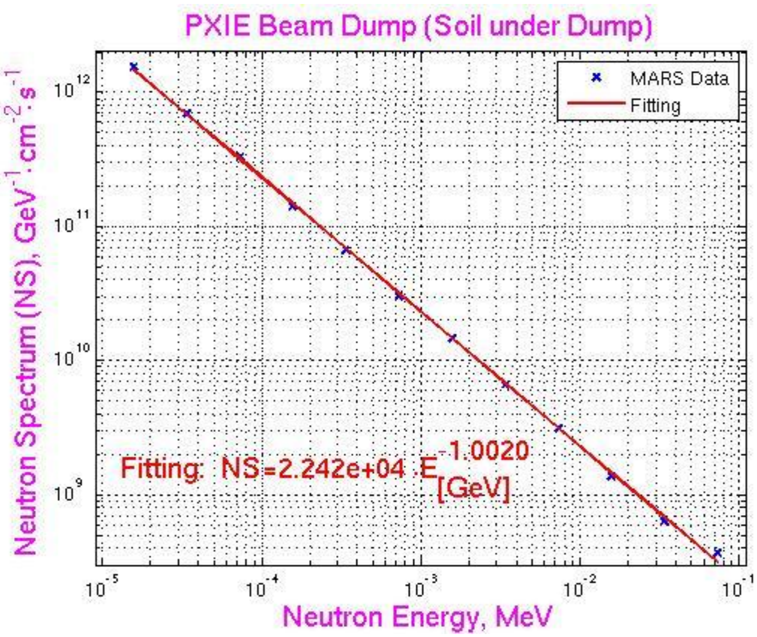

Figure 10: Spectral neutron distribution and its linear interpolation.

The result of the linear interpolation shows that to a good accuracy the neutron spectrum is inversely proportional to the energy of neutrons:

$$\frac{dN_n}{dE} = A_{spectrum} \cdot E_{[GeV]}^{-\alpha_{spectrum}}, \text{ where } A_{spectrum} = 2.242 \cdot 10^4 \ GeV^{-1} \cdot cm^{-2} \cdot s^{-1} \ \text{ and } \ \alpha_{spectrum} = 1.0002.$$

Integrating data of Figures 7 and 10 over the neutron energy (good quality of the interpolation allows do it analytically) one obtains the rate of tritium production $Q_{[atom/(cm^3 \cdot s)]}$ to be equal to:

$$Q_T = N_{^6Li_3} \cdot \int_{E_{\min}}^{E_{\max}} \sigma_{Li}(E) \frac{dN_n(E)}{dE} dE = N_{^6Li_3} \cdot A_{Li} \cdot A_{spectrum} \int_{E_{\min}}^{E_{\max}} E^{-(\alpha_{Li}+\alpha_{spectrum})} dE =$$

$$= N_{^6Li_3} \cdot A_{Li} \cdot A_{spectrum} \frac{E_{\max}^{1-(\alpha_{Li}+\alpha_{spectrum})} - E_{\min}^{1-(\alpha_{Li}+\alpha_{spectrum})}}{1-(\alpha_{Li}+\alpha_{spectrum})}.$$

Next Figures show the dependence of tritium production on selection of $E_{\min}$ and $E_{\max}$.

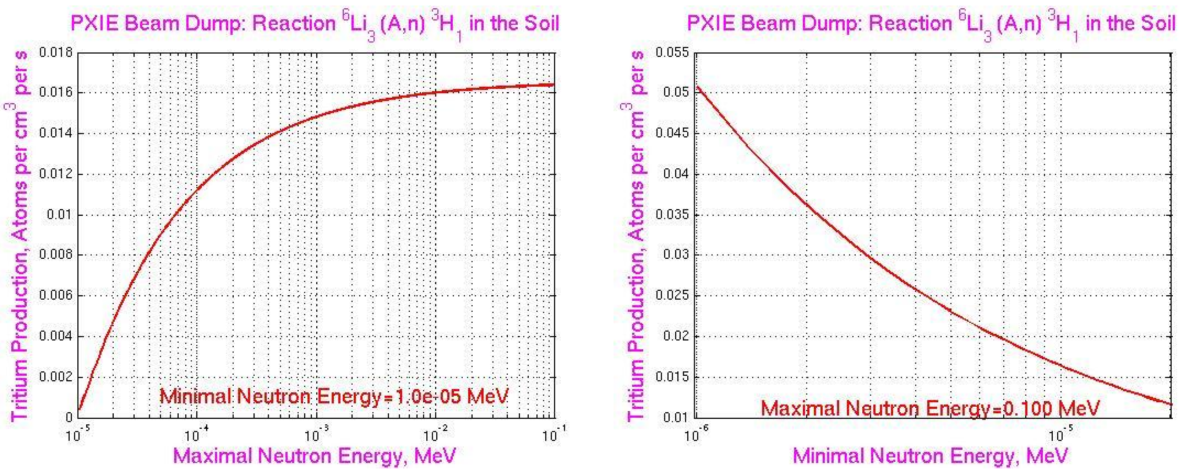

Figure 11: Tritium production vs $E_{\max}$ an $E_{\min}$.

One can see that the integral has weak dependence on $E_{\max}$ for neutron energies above $E_{\max} = 10^{-5} MeV$. Unfortunately, there are no data of the cross section of the reaction $^6\mathrm{Li}_3(n,\mathrm{A})^3\mathrm{H}_1$ for neutrons with energies below $E_{\min} = 10^{-5} MeV$. Extrapolation of the energy dependence in the area below $E_{\min}$ allows evaluate the effect of the lower threshold energy (Fig. 11). Eventually, interval $10^{-5} \le E \le 10^{-1} \ MeV$ is used for subsequent evaluations.

The equation of the time dependence concentration of tritium is

$$\frac{dN_T}{dT} = -\lambda N_T + Q_T,$$

where $\tau_{1/2} = \log 2 / \lambda = 12.35$ year is half-life if tritium. Solution of this equation is

$$N(t) = \frac{Q_T}{\lambda}\left(1 - e^{-\lambda t}\right) = \frac{Q_T \cdot \tau_{1/2}}{\log 2}\left(1 - 2^{-t/\tau_{1/2}}\right).$$

Let estimated time of the PXIE operation is $\tau_{PXIE} \approx 5$ year. After that the accumulated amount of tritium will only decrease due to radioactive decay. Next figure shows the accumulation of the tritium in the soil under the PXIE beam dump with life time of the PXIE taking into account.

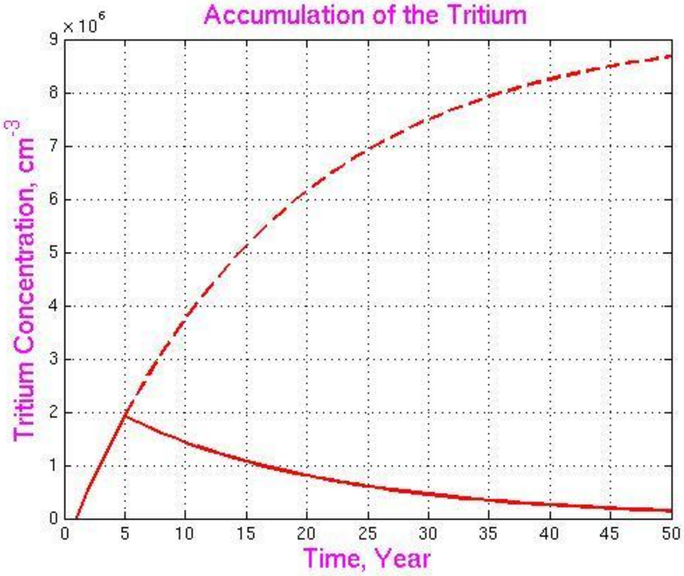

Figure 12: Accumulation of the tritium under PXIE beam dump. Dashed line – case with $\tau_{PXIE} = 50$ year.

Now it is very simple to find the activity of the tritium: $Y_T = f\lambda N_T = f \cdot \frac{\log 2}{\tau_{1/2}} \cdot N_T$, where $f = 1/0.037$ pCi/(disintegration per s) is the conversion factor. Figure 13 shows the time dependence of tritium activity.

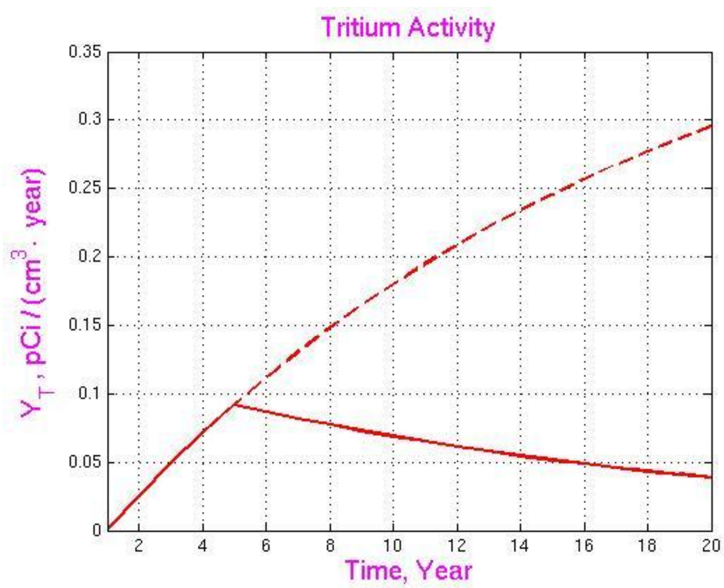

Figure 13: Activity of the tritium under PXIE beam dump. Dashed line – case with $\tau_{PXIE} = 50$ year.

Thus, the maximal tritium yield during first 5 year of the PXIE operation is $Y_T \le 0.1\ \mathrm{pCi/(cm^3 \cdot year)}$. This value is significantly below than the level of $20\ \mathrm{pCi/(cm^3 \cdot year)}$ for tritium activity in the soil allowed by Fermilab safety manual.

Conclusions

An estimate of tritium production in the soil under the PXIE absorber yields significantly smaller value than the maximum acceptable value determined by Fermilab safety regulations.

References

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