

## Analysis of magnetic measurements of the solenoidal lens # PSSA003

### Introduction

We are preparing specifications for the solenoidal lenses of PXIE LEBT. One of the questions is whether we should specify a requirement on the lens field quality. The natural measure of the aberrations in such lenses is the spherical aberration, and it would be useful to find a set of measurements that would ensure that other, non-axially-symmetric focusing errors are significantly lower.

In attempt to understand possible aberrations in real high-field solenoidal lenses, I looked at the measurement data of a similar lens recently (2011) designed and manufactured in TD (Vl. Kashikhin, A. Makarov) for the LEBT of the Proton Improvement Plan. This note assembles several plots from MathCad analysis that were useful for my own education.

The main conclusion is that no reliable indications of high-order components are found in this lens, but the dipole kick by the lens can be noticeable.

### Data

Files `ptscan_raw_pnts.4664538` and `ptscan_raw_pnts.4666935` contain three components of magnetic fields measured with 3D Hall probe assembly in 2011 at a cylindrical surface of 1" radius and length 10.5" with the axial step of 0.5" and 24 point azimuthally at currents of 100,200...500 A. the units in the files are Tesla and inches. The data files for the lens # PSSA003 were provided by Dana Walbridge.

The analysis was made in MathCad. The reported shift of the Hall probes with respect to each other was not taken into account.

Fig.1 shows a typical longitudinal field distribution.

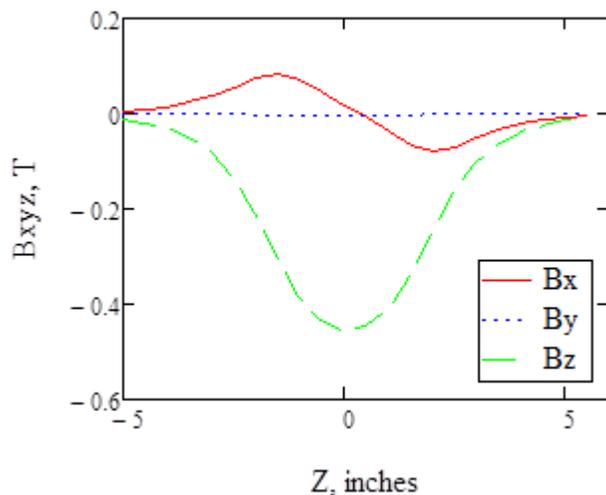


Figure 1. Example of the longitudinal distribution of field components at  $X=1''$ ,  $Y=0$ ,  $I_{\text{coil}} = 500\text{A}$ .

To get an idea about accuracy of measurements, deviations of the integral  $\int B_z dz$  over azimuth were calculated and found to be  $<0.5\%$  for  $I_{\text{coil}} = 500\text{A}$  (Fig.2). Number of turns calculated for each current from the average over the azimuth is shown in Fig. 3. The points are within  $0.3\%$ . The trend may be related to the field leakage out of the measurement range related to partial saturation of the yoke.

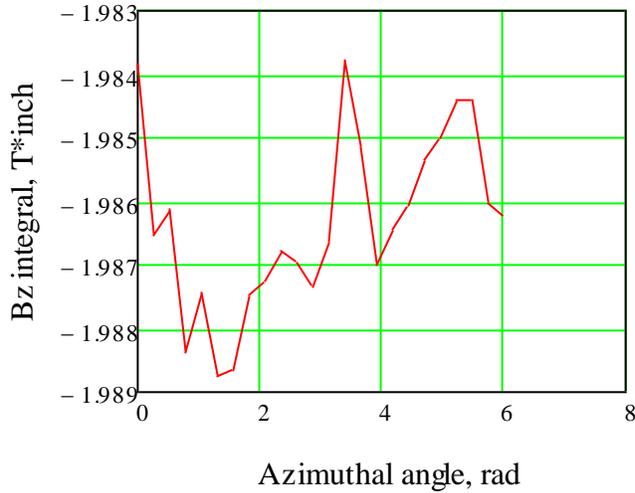


Figure 2. Integral  $\int B_z dz$  as a function of the azimuthal angle.  $I_{\text{coil}} = 500\text{A}$ .

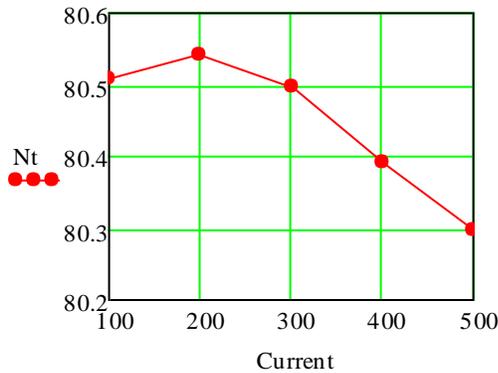


Figure 3. Number of turns in the coils calculated from  $\int B_z dz$  integral for different currents.

Figure 4 shows the azimuthal deviations of the  $\int B_z^2 dz$  as a function of the azimuthal angle. Within the measurement scatter, it can be explained by a vertical shift of the probe by 2 mm (fitting curve in Fig.4).

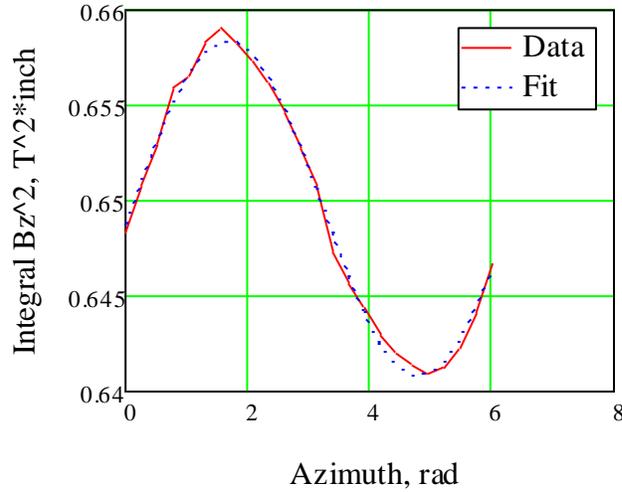


Figure 4. Integral  $\int B_z^2 dz$  as a function of the azimuthal angle.  $I_{\text{coil}} = 500\text{A}$ .

The longitudinal distribution  $B_z(z)$  can be fitted by (e.g. as in OptiM [1])

$$B_z(z) = \frac{B_0}{2} \left[ \tanh\left(\frac{z + 0.5L - z_c}{a}\right) - \tanh\left(\frac{z - 0.5L - z_c}{a}\right) \right] \quad (1)$$

Fitting is shown in Fig. 5 for  $B_0 = -0.578\text{T}$ ,  $L = 3.43''$ ,  $a = 1.58''$ ,  $z_c = 0.09''$ .

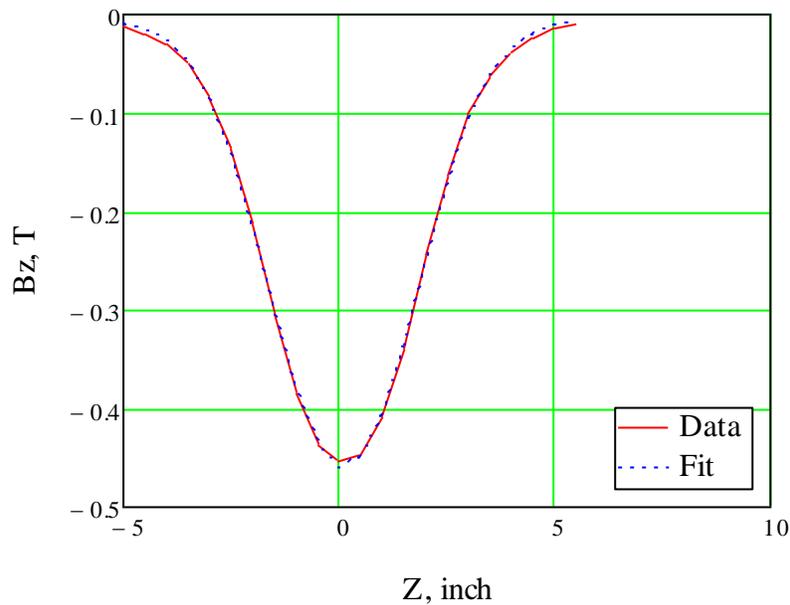


Figure 5. The longitudinal distribution of  $B_z$  field component at  $X=1''$ ,  $Y=0$ ,  $I_{\text{coil}} = 500\text{A}$ .

The spherical aberration estimated by the formula suggested by S. Nagaitsev [2] gives deviation of the  $\int B_z^2 dz$  integral at 1" offset by 8.4%. Comparison of the average of this set (at  $I_{\text{coil}} = 400\text{A}$ ) with the field measured on axis (data supplied by M. Tartaglia) gives 7.1%, which looks like a reasonable agreement.

One more check of asymmetry was made by integrating the total transverse component  $\sqrt{B_x^2 + B_y^2}$  from  $Z=-5''$  to  $Z=0$ . The deviation from average over the azimuth is shown in Fig. 6 as well as a correction that assumes the shift and its phase found in fitting of Fig.4. While the agreement is not as good as in Fig.4, the deviation related to the shift is dominant. Note that the shifts and angle of individual Hall probes may affect this type of measurements.

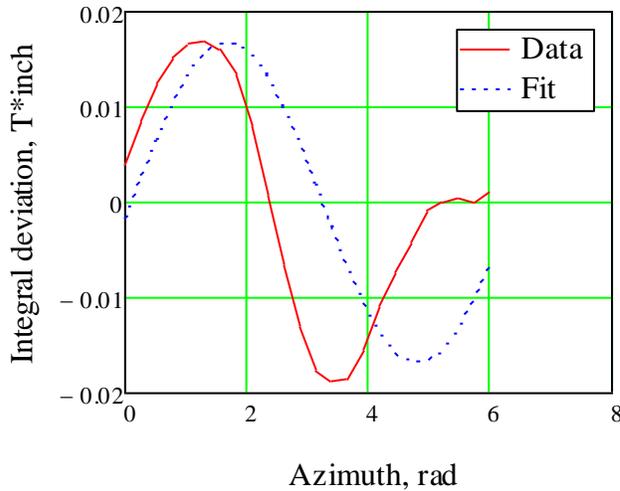


Figure 6. Deviation of the integral  $\int_{-5}^0 \sqrt{B_x^2 + B_y^2} dz$  from its average as a function of azimuthal angle.  $I_{\text{coil}} = 500\text{A}$ . In the same units, the average of the integral is 0.208 T·inch.

A separate concern is dipole fields. Figure 7 shows integrals  $\int B_x dz$  and  $\int B_y dz$  for  $I_{\text{coil}} = 500\text{A}$ .

There is no strong dependence on azimuth. The fields correspond to the average angle  $\frac{\int B_x dz}{\int B_z dz}$

~10 mrad for each X and Y components. The X angle increases from 6 to 11 mrad at the increase of the lens current from 100 to 500A, while the Y angle changes much less significantly, from 7.2 to 7.8 mrad.

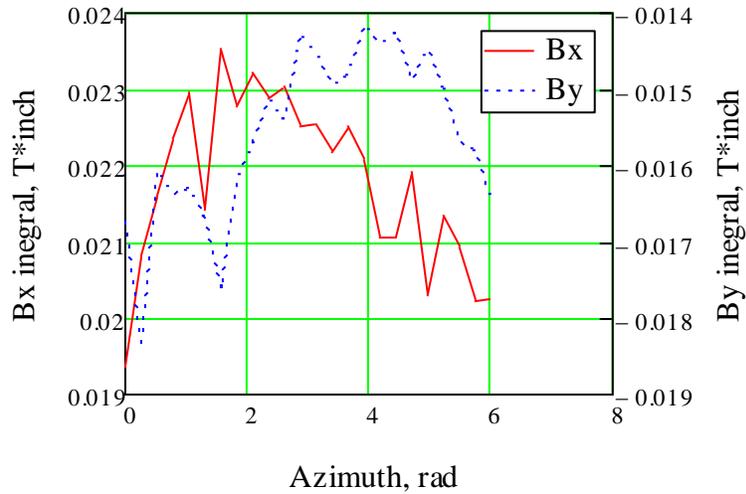


Figure 7. Integrals of the transverse field as functions of azimuth.  $I_{\text{coil}} = 500\text{A}$ .

To estimate the scale of the effect of the dipole fields, Fig. 8 shows angles acquired by a 30 keV proton after passing through the lens on axis (assuming that the lens axis to be the line  $X=Y=0$  in the measurements). The X angle is noticeably nonlinear. The total angle reaches 27 mrad at  $I_{\text{coil}} = 500\text{A}$ .

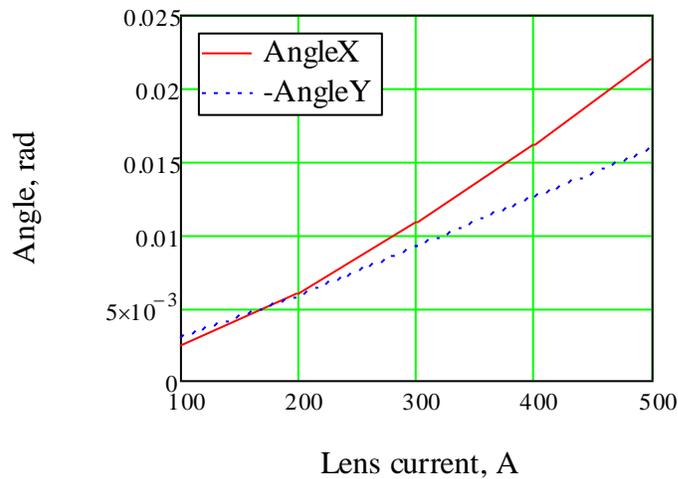


Figure 8. The angles imposed on a 30 keV proton moving through the lens on axis.

### Conclusion

1. The measured spherical aberration agrees with an analytical estimation.
2. There is no reliable evidence of components that would result in non-axially-symmetrical focusing by the lens. Observed deviations can be explained (and, in practice, compensated) by a 2 mm shift of the lens with respect to the axis of measurements.

Remaining deviations from axial symmetry are lower than the spherical aberration contribution (likely, much lower).

3. There is a significant dipole component in the lens field; the average dipole field creates ~10 mrad angle of the field with the mechanical axis (assuming that the measurements were made along the mechanical axis). If, for example the lens were used for PXIE LEBT's 30 keV H- at the maximum field, the axial particles would be kicked off by 27 mrad.
4. Taking into account uncertainty in the Hall probe positioning, I do not see a simple measurement that would ensure a low value of non-axially-symmetric components.

I am thankful to D. Walbridge and M. Tartaglia for the data.

#### References

1. OptiM - V. Lebedev, A. Bogacz, "Betatron motion with coupling", JLAB-ACC-99-19 (1999). Available at <http://www-bdnew.fnal.gov/pbar/organizationalchart/lebedev/OptiM/optim.htm>
2. L.Prost et al., Estimations of spherical aberrations for PXIE LEBT solenoids, <http://projectx-docdb.fnal.gov/cgi-bin/ShowDocument?docid=1015>